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On the robustness of the competitive equilibrium: Utility-improvements and equilibrium points ☆

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ABSTRACT

We show that the set of competitive equilibrium points of a pure exchange economy are the equilibrium points of a broader class of better-response demands than the usual utility-maximizing demand functions. The better-response demands are derived from assigning weights to all commodity bundles with higher utility than the current commodity bundle, with the greatest weights being placed on the commodity bundles with the highest utility gain. The usual utility-maximizing demand functions are then those in which the weight on the utility-maximizing bundle is one. We also show that these better-response demands belong to a large class of response maps that are generated by monotonic transformations of the utility functions and/or monotonic transformations of the weights assigned to the commodity bundles.
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1. Introduction

As is well known, the competitive equilibrium or the Walrasian equilibrium prices are prices at which the market clears (that is, prices at which the excess demand is zero) when economic agents respond to market prices by choosing the utility maximizing bundle of goods and services. The inference that can then be made is that the competitive equilibrium allocation of goods and services can prevail only if all agents trade their optimal or utility maximizing bundle at the market prices. Little has been said about what would happen if some of the economic agents respond to market prices by demanding bundles different from the utility-maximizing bundles. One is then led to conclude that in such cases the market would be in disequilibrium, or, if the deviations from utility maximizing behavior are not too large then trade would take place at approximate equilibrium market prices.

What we show here is that if the economic agents use a demand function that belongs to a class of better-response demand functions, then the market clearing prices are exactly the same as the competitive equilibrium prices. More precisely we show that if consumer i offers to trade commodity bundles like

$$d_i(x^i, p) = \frac{m^i(x^i) + \int_{\bar{\gamma}_i(p,p\cdot\omega^i)} \phi^i(x^i, a)a\mathrm{d}a}{1 + \int_{\bar{\gamma}_i(p,p\cdot\omega^i)} \phi^i(x^i, a)\mathrm{d}a}$$

when the market price is p and the preceding commodity bundle offered for trade is x^i, then the resulting market clearing prices are the competitive equilibrium prices. Here ω^i is the endowment vector of agent i and $\bar{\gamma}_i(p, p\cdot\omega^i)$ is the tradable boundary of the budget set of consumer i given the endowment ω^i and the price vector p. $m^i(x^i)$ is the bundle in $\bar{\gamma}_i(p, p\cdot\omega^i)$ that is closest to the bundle x^i, and the function

$$\phi^i(x^i, a) = \max\{0, [u^i(a) - u^i(x^i)]\}$$

is the gain in utility from bundle a over bundle x^i. Note that this demand function can be written as

$$d_i(x^i, p) = \beta(m^i(x^i))m^i(x^i) + \int_{\bar{\gamma}_i(p,p\cdot\omega^i)} \beta(x^i, a)a\mathrm{d}a,$$

that is, as a weighted average of the bundle $m^i(x^i)$ and the bundles that have higher utilities than x^i, where the weight on bundle a is

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$\beta(x^i, a) = \frac{\phi^i(x^i,a)}{1+\int_{\bar{\gamma}_i(p,p\cdot\omega^i)}\phi^i(x^i,a)\mathrm{d}a}$ and the weight on the bundle $m^i(x^i)$ is $\beta(m^i(x^i)) = \frac{1}{1+\int_{\bar{\gamma}_i(p,p\cdot\omega^i)}\phi^i(x^i,a)\mathrm{d}a}$. The first thing to note about this better-response demand function d_i is that it is not the consumer's utility maximizing bundle, but only a weighted average of all the bundles that are in the boundary of the budget set $\bar{\gamma}_i(p, p\cdot\omega^i)$ of the consumer and which have higher utilities than the bundle $m^i(x^i)$. The other significant feature of this demand function is that the weights $\beta(x^i, a)$ of the commodity bundles a that are preferred to x^i increase with the utility gain from the bundles a, the largest weight being on the utility-maximizing bundle.

One interpretation of these weights could be that the weights are the probabilities with which an agent chooses a particular bundle. As the weights are larger for commodity bundles with higher utility gains, a bundle with a higher utility gain has a greater probability of being chosen. That an agent does not immediately choose a utility maximizing bundle may then be due to errors that the agent makes in choosing the commodity bundle. Since the probability of choosing a commodity bundle with a larger utility gain is higher, an agent then makes a *less costly* error (that is, a smaller loss in utility from the maximum possible) with a higher probability than a *more costly* error (that is, a larger loss in utility from the maximum possible). Thus, although economic agents may be prone to making errors, they make more costly errors with a smaller probability and less costly errors with a higher probability.[1]

It is useful to note here that the result presented here is quite different from the observation that *perturbations* of the utility maximizing choices give us *approximate* equilibrium points, and that as the perturbations go to zero, the approximate equilibrium points converge to exact equilibrium points; where such convergence is guaranteed by the upper semi-continuity of the equilibrium correspondence. The equilibrium points resulting from consumers using the better-response demand functions $d_i(x^i, p)$ are exact equilibrium points. This is thus distinct from studies, mostly in game theory, that look at the outcomes of games when players do not fully maximize utility because of bounded rationality. In McKelvey and Palfrey (1995) and McKelvey and Palfrey (2009), players observe utilities with some errors which can then affect their utility maximizing choice; the error in observing the true payoff can lead to non-utility maximizing choices. Such choices can lead to payoffs that are actually less than the current payoff of the player. In Chen et al. (1997) players retain a level of bounded rationality and never quote a utility maximizing bundle unless the bounded rationality goes to zero. An alternative approach to bounded rationality is to use the concept of control costs as in Mattson and Weibull (2002). It is another explanation as to why a decision maker may not necessarily quote his or her utility maximizing bundle. In all these alternative approaches the "equilibrium" could be different from the equilibrium obtained when players or agents maximize utility. In our case, the equilibrium is the utility-maximizing equilibrium. This is so because in our case players or agents get to make better choices until an equilibrium is reached. Consumers (or agents or players), even when choosing a non-utility maximizing bundle, never make a choice that is "less" preferred than their current choice.

A point worth noting about the better-response demand functions d_i is that these demand functions seem to describe behavior that is distinct from just satisficing. An example of a demand function that would indicate that agents were just satisficing is one in which an agent takes a *simple* average of all the bundles that are better than the one currently held by the agent. Demand functions generated by such satisficing behavior do not, in general, give equilibrium points that are competitive equilibrium points.[2] The difference between behavior that is represented by the better-response demand functions d_i and general satisficing behavior is that the demand functions d_i result from agents placing increasingly larger weights on bundles with higher utility gains. In fact, as we show, any system of weighting of the commodity bundles that are monotonically increasing in the weights $\beta(x^i, a)$ will generate demand functions that result in equilibrium points that are competitive equilibrium points.

Better-response functions like the better-response demand functions being discussed here have been used in the game theory literature. In proving the existence of a Nash equilibrium in Nash (1950a) Nash used a better-response function distinct from the *best response* map that appears in Nash (1950b). Nash (1951) offered a refined version of the existence result in Nash (1950a) that used the Brouwer Fixed Point Theorem and showed that the *better response* map's fixed points are Nash equilibria, and vice versa. More recently, Becker and Chakrabarti (2005) showed that Nash's better-response function belongs to a class of better-response maps whose fixed points are Nash equilibrium points and extend the results in Nash (1950a) and Nash (1951) to games with a continuum of actions and to games with some forms of non-expected utilities.[3] The interesting feature of the map that is used by Nash in Nash (1950a, 1951), and which is used in Becker and Chakrabarti (2005), is that this map is a weighted average of the responses that are better than the current strategy; the weights being increasing functions of the gains in payoffs. This better-response function, adapted to the environment of a market and to that of the abstract economy associated with that market, gives the better-response demand functions that we discuss here.

As we know, in his seminal papers on the existence of a competitive equilibrium, Debreu (1952) uses the concept of an abstract economy and social equilibrium to establish the existence of a competitive equilibrium.[4] The abstract economy is a pseudo-game in which the choice sets of the participants depends on the choices made by all the participants, unlike that in a game in which the choice sets are independent of the actions of the players. Results for an abstract economy can thus be used to establish results for games and conversely.

We use these observations and the fact that there is a close connection between the social equilibrium of the abstract economy and the competitive equilibrium to show that the competitive equilibria are the rest points of a market process in which the demand functions are not necessarily the utility-maximizing bundles, but better-response demand functions that are the weighted average of all those commodity bundles that have higher utilities than the current bundle. Thus if buyers and sellers respond to the prevailing price so as to buy or sell a basket of goods that is "better" than the one they have, but not necessarily the best, primarily because they may have made errors in choosing the utility maximizing bundle, then not only is there some kind of an equilibrium, but it happens to be the competitive equilibrium. This would then

[1] This interpretation of how agents respond echoes some of the observations made in game theory. For example, trembling hand perfect equilibrium (Selten, 1975) considers equilibrium points that are robust to "trembles" or errors made by the agents. The concept of "proper" equilibrium (Myerson, 1978) is even more closely connected with our interpretation here, as it requires equilibrium points to be robust to trembles in which players make more costly errors with lower probabilities and less costly errors with higher probabilities.

[2] Such demand functions can be discontinuous and may not generate any equilibrium points at all.

[3] Nash proves the existence of an equilibrium for games with finite actions and expected utilities.

[4] Debreu (1952) uses a generalized version of Nash's existence theorem to prove the existence of a social equilibrium and shows that a social equilibrium of a properly defined abstract economy is a competitive equilibrium.

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