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Dynamic theory of preferences: Habit formation and taste for variety☆

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ABSTRACT

We analyze individual preferences over infinite horizon consumption choices. Our axioms provide the foundation for a recursive representation of the utility function that contains as particular cases the classical Koopmans representation (Koopmans (1960)) as well as the habit formation specification.

We examine some of the consequences of our axiomatization by considering a standard consumer choice problem, and show that typically in the space of concave utility functions satisfying our axioms the consumer displays a taste for variety. The latter means that such a consumer selects optimally time variant consumption programs for any given time invariant sequence of commodities' relative prices and for all possible sequences of market discount factors. In contrast, if a concave utility function satisfies Koopmans' axioms the consumer does not display a taste for variety.

1. Introduction

This paper provides an axiomatic foundation for preferences over infinite horizon consumption choice problems where memory of past consumption experience affects preferences over present and future choices. The individual preferences we consider are defined over past, present and future consumption bundles. Our axioms deliver a recursive representation of the utility function that contains as a particular case the habit formation specification as well as the classical Koopmans' theory of inter-temporal preferences (Koopmans, 1960).

Typically, decision theory imposes axioms on preferences over a given set of choices which are available to the decision maker. Here, we deviate from this approach by imposing axioms over a larger set. Our consumption set is the set of consumption paths ranging from the infinite past to the infinite future; past consumption paths are of course not choice variables of the consumer. There are instances in economic theory where preferences over choices depend on other variables. This is, for instance, the case for the habit formation model where preferences over present and future consumption paths are affected by past consumptions. By imposing axioms on our larger consumption set, we give a systematic order to this dependence.

There are two good reasons that justify this research. First, it provides an axiomatic foundation for the habit persistence specification of the utility function. Macro and finance modeling has made an extensive use of such utility functions, (beginning with Constantinides, 1990 and Sundaresan, 1989). Such modeling choice has been able to provide an explanation for the asset return puzzles and to improve the ability of the standard real business cycle model to replicate real data, e.g., Boldrin et al. (2001). Habit persistence requires the utility function to be time separable, and to depend on present consumption and on a habit stock. Since the latter is a function of the past consumption choices, the felicity index depends on present as well as on past consumption.

The second motivation for the reason presented here is to provide an axiomatic treatment for individual preferences over infinite horizon consumption plans that allows for optimal consumer programs to be time variant even in otherwise stationary environments. We use the notion of *taste for variety* to denote a time dependence of the optimal path which is generated not by the environment, but by the underlying individual preferences. This feature does not have to be confused with amplification of the time variability of consumption paths. For instance, the standard additive, separable specification of the utility function produces, in an environment subject to shocks, more path variability than the habit formation specification. Indeed, it is the persistence of the shocks one of the ingredients explaining the success of the habit formation

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specification. However, the standard additive separable case produces constant optimal paths in stationary environments where shocks are absent. This is a constraint that we like to relax.

The recent literature on time preferences has focused on issues some what extraneous to this paper. Starting from the study of the time realization problem (Fishburn and Rubistein, 1982), the work has rediscovered time inconsistency issues originally formulated in Strotz (1956) and later on in Laibson (1997) and it has moved towards general representation encompassing different experimental and theoretical models, Ok and Masatlioglu (2004) and Dubra (2004).

1.1. The axiomatic analysis

Standard theory of convex preferences rules out a taste for variety. It does so by ruling out two different possible sources of preference for variety.

The first is a *static* taste for variety. This is ruled out by the assumption of convexity of preferences. A static taste for variety in infinite horizon consumption allocation problems is allowed by a time separable utility function, when the utility function of each period is non concave.

The second, a *dynamic* taste for variety, is ruled out in the theory of inter-temporal preferences as in Koopmans (1960) by the two *limited complementarity* assumptions (see Postulate 3a and 3b Koopmans, 1960). The first of them requires preferences over today's consumption bundles to be independent of any continuation path. The second requires the preferences over future consumption paths to be independent of the commodity bundle consumed today. It is this last axiom that makes the habit formation specification of the utility function incompatible with Koopmans' construction. With habit formation, present and past consumptions choices affect the habit stock and, hence, the preferences over continuation paths. This contradicts the second *limited complementarity* axiom of Koopmans which eliminates exactly this possibility. In our setting of the problem and the basic axioms we relax this requirement, and thus allow for preferences that may exhibit habit formation. Thus our setup is one which is consistent with habit formation. An axiom system producing specific forms of habit formation, as in the applied literature we recalled, is open for future research.

Koopmans also rules out *time inconsistency* by imposing a *stationarity* axiom. Stationarity requires that the preference ordering over consumption program be the same as the ordering over corresponding "*shifted*" programs, i.e., programs obtained by advancing the time of each future consumption vector by one period. Once stationarity is assumed, the second limited complementarity axiom becomes redundant. An equivalent way of stating this is that the two central axioms of Koopmans construction require first preferences to be stationary and, second, preferences over present consumption bundles to be independent of future continuation paths.

In our construction, a consumption program is a sequence of consumption bundles ranging from the infinite past to the infinite future. Thus, the preferences over present and future consumption bundles may depend on the past consumption profile experienced by the individual. We impose a set of axioms that weakens those chosen by Koopmans (1960). If we restricted individual preferences to be independent of past histories, we would get back exactly Koopmans construction. We adapt the Koopmans' first limited complementarity axioms to our structure by requiring that the future continuation consumption paths do not alter the ordering over present–past consumption paths. We do not impose a limited complementarity axiom over the shifted programs since the latter is implied by the stationarity axiom, that we are going to impose. Stationarity rules out time inconsistency and it requires, as in Koopmans, that the ordering over programs and shifted programs be identical. However, while we impose stationarity as we move the time (following its physical interpretation) forward, we do not impose it backward. Thus, if the time flow could be reversed our preferences would be (backward) time inconsistent.

We get as Koopmans a representation theorem yielding an aggregator expressing the utility function over consumption programs as a function of two different utilities. The first is the due generalization of Koopmans' *immediate utility*, which in our case is the utility generated by the path of past and present consumptions. The second is the Koopmans' *prospective utility*, the utility generated by the shifted program. Furthermore, the stationarity axioms guarantees that the prospective utility functions and the utility functions are identical. As for the Koopmans construction, we have to impose technical axioms that require continuity and rationality of the preference relation. The nature of these axioms is technical and it is discussed in the body of the paper.

1.2. Taste for variety

In the second part of the paper, we study the implications of Koopmans and our preference axioms on the *taste for variety*. The consumption set of our consumer is at each period a subset of the positive cone of a C-dimensional Euclidean space, (with C greater than one). The individual is endowed with positive wealth, faces a unique budget constraint and a sequence of time invariant relative prices. A consumer reveals a taste for variety if for all sequences of market discount factors, the optimal path is time variant. Otherwise, she does not. To avoid a trivial problem we require convexity of individual preferences. We take the consumer problem so far described as the prototypical example of a stationary environment.

Koopmans axioms are not sufficient to generate a time additive and separable utility functions (Koopmans, 1972a,b). However, we show that a consumer with such preferences does not display a revealed taste for variety and that the market discount factors supporting the time invariant consumption program are, as for the time additive, separable case, time invariant.

In our world, the analysis of the revealed taste for variety is more complex. Preferences over present and future consumption are now defined up to an "initial condition" specifying the past consumption path. Thus, the optimal solution to the consumer problem typically depends on such an arbitrarily given initial condition. A consumer reveals a *weak revealed taste for variety* if the optimal program displays time dependence for all sequences of market discount factors and some given initial conditions. It is always possible to find an initial condition delivering for some sequence of market discount rates a constant optimal path. This is an immediate consequence of our representation theorem and of the assumed convexity of preferences. On the other hand, we show that for typical choices of pairs of initial conditions and utility functions, our consumer displays a revealed taste for variety. However, we do not find this result sufficiently convincing as we find questionable whether or not in a stationary environment initial conditions can be treated as free parameters of the consumer problem as such suitable for genericity statements. In order to discuss this issue, we analyze a different consumer problem where choices are elements of our consumption set, that is of the set of consumption sequences ranging from the infinite past to the infinite future.

A consumer reveals a *strong revealed taste for variety* if the optimal path is time variant for any sequence of market discount rates ranging from the infinite past to the infinite future. This strong notion contradicts the essential nature of time, but its only purpose is to give insights for the choice of an initial condition. Indeed, suppose that a utility function does not satisfy the strong definition, but satisfies the weak definition for some past histories. In this