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Equilibrium in collateralized asset markets: Credit contractions and negative equity loans

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ABSTRACT

We address a general equilibrium model with collateralized debt, credit contractions, and financial market segmentation. Restrictions on credit access make borrower's optimal payment strategies – coupon payment, prepayment, and default – sensitive to idiosyncratic factors, even though the only payment enforcement is the seizure of collateral guarantees. We prove equilibrium existence, characterize optimal borrower's payment strategies, and provide a numerical example illustrating our main results. A remarkable feature of our model is that it rationalizes the prevalence of negative equity non-recourse loans.

1. Introduction

As it is well known, collateralized borrowing plays a central role in financial markets. With the aim of better understanding collateralized asset markets, we consider a general equilibrium model with non-recourse collateralized loans, credit contractions, heterogeneous access to financial instruments, and long-lived loans. We capture the three most relevant risk factors underlying asset-backed security markets: credit risk, prepayment risk, and interest rate risk. Also, in our model credit contractions and financial market segmentation make borrower's optimal payment strategies (i.e., coupon payment, prepayment, and default) sensitive to idiosyncratic factors.[1] That is, in contrast with previous general equilibrium approximations to collateralized borrowing, creditors do not necessarily follow a homogeneous action delivering the minimum between the amount of debt and the market value of the associated collateral. An interesting consequence of the assumed financial structure is that it rationalizes the prevalence of negative equity loans (i.e., debts with a lower collateral value than the associated prepayment cost), even in the absence of additional enforcement mechanisms.

We extend the seminal model of Geanakoplos and Zame (2013) to a three-period setting with long-lived securities.[2] In our model, each credit contract is characterized by its emission node, coupon payments, prepayment rule, and collateral requirements.[3] After the emission of a credit contract, borrowers have the possibility to pay the coupon or close short positions by either delivering the collateral or prepaying. Our model captures credit contractions as an exogenous variation of the set of existing debt instruments and market segmentation through financial participation constraints. Coupon and prepayment rules are just required to be continuous functions of prices and, therefore, a wide variety of characterizations of debt contracts is possible.

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1 As emphasized by Deng et al. (2000), idiosyncratic components are essential to estimate default and prepayment risks.

2 The model in Geanakoplos and Zame (2013) first appeared in several working paper versions (see Geanakoplos and Zame (1997, 2002, 2007)). Their model has previously been extended by Araujo et al. (2005b, 2011) to include long-lived securities in an infinite-horizon framework. As pointed out above, a major divergence of our model with respect to these latter works is the incorporation of credit contractions and financial market segmentation. These frictions allow us to capture heterogeneous optimal payment strategies.

3 Although in our model the only payment enforcement is the seizure of collateral guarantees, different enforcement mechanisms have been considered in the literature of general equilibrium addressing credit risk. The effect of utility penalties has been analyzed by Dubey et al. (1989, 2005), Zame (1993), Páscoa and Seghir (2009), and Martins-da-Rocha and Vailakis (2012a,b); default dependent participation constraints have been considered by Kehoe and Levine (1993), Kocherlakota (1996), and Alvarez and Jermann (2000); bankruptcy mechanisms have been considered by Araujo and Páscoa (2002), Sabarwal (2003), and Poblete-Cazenave and Torres-Martínez (2013).

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To preserve the anonymity of trading in the presence of heterogeneous optimal payment strategies we follow Dubey et al. (2005) assuming that debt payments are pooled into pass-through securities. Each security associated with a credit contract provides endogenous payments with match aggregate borrowers' deliveries in equilibrium. Investment opportunities vary endogenously as these depend on default and prepayment decisions. Therefore, financial markets could become more incomplete as a consequence of debtors' decisions.

The consideration of credit contractions and financial market segmentation is crucial to have prepayment and credit risks affected by idiosyncratic characteristics. These frictions make borrowers more willing to maintain short positions, even negative equity positions. Indeed, in a version of our model without credit tightening borrowers would always give strategic default, closing negative equity loans.

The prevalence of negative equity non-recourse loans is an empirically observed pattern in a significant fraction of mortgage loans in the US. In the second quarter of 2013 over 14% (around 7.1 million) of mortgage loans were in negative equity,[4] a figure that has decreased from 24% in the fourth quarter of 2009. Additionally, although there are significant differences in the share of negative equity loans across states, it is particularly relevant that in some US non-recourse states, e.g. Arizona and California, the share of mortgage loans in negative equity is substantially above the aggregate figure.[5]

A non-arbitrage analysis of individually optimal payment strategies reveals that some agents could optimally decide to continue paying negative equity loans. However, the existence of more attractive credit opportunities – in terms of downpayment and interest rates – may trigger agents' decision to close short positions. Hence, agents could optimally decide to maintain underwater loans as a response to credit contractions. This result identifies a novel channel which rationalizes the persistence of negative equity non-recourse mortgages in the absence of additional payment enforcements. This channel helps to understand why most borrowers with negative equity mortgages (84.8% at the end of the first quarter in 2012) are honoring their commitments in a period of a significant tightening of credit standards.

We provide a numerical example illustrating all possible payment strategies in our model: payment, prepayment, and default. We show that different agents may adopt different optimal payment decisions and discuss the effect of credit contractions and market segmentation on these decisions. In particular, it is shown that underwater mortgages are a possible equilibrium outcome.

The rest of the paper proceeds as follows: Section 2 sets out the model, notation and equilibrium definition, Section 3 specifies several types of coupon and prepayment rules captured by our model, Section 4 establishes equilibrium existence, Section 5 characterizes optimal payment strategies following a non-arbitrage analysis, Section 6 contains a numerical example, and Section 7 provides some concluding remarks. All the proofs are left to an Appendix.

2. The model

Information structure. We consider a dynamic economy $\mathcal{E}$ with three periods, $t \in \{0, 1, 2\}$. There is uncertainty about the state of nature that will be realized, which belongs to a finite set S. The information available at period t is symmetric and given by a partition $\mathbb{F}_t$ of S, with $\mathbb{F}_0 = \{S\}$, $\mathbb{F}_1$ at least as fine as $\mathbb{F}_0$, and $\mathbb{F}_2 = \{\{s\} : s \in S\}$. Hence, economic agents are uninformed at the first period, information increases through time, and they become perfectly informed in the last period.

Let D be the event-tree composed of nodes $\xi = (t_\xi, \sigma_\xi)$, where $t_\xi \in \{0, 1, 2\}$ and $\sigma_\xi \in \mathbb{F}_t$. Denote by ξ_0 the unique initial node and by D_t the set of nodes dated t. A node μ is a successor of ξ, referred as $\mu > \xi$, when both $t_\mu > t_\xi$ and $\sigma_\mu \subseteq \sigma_\xi$. Thus, given a node ξ, we denote by ξ^- its immediate predecessor and by ξ^+ the set of immediate successors.

Physical markets. There is a finite and ordered set of commodities, L, which are traded in spot markets and may experience transformations between periods. Hence, a bundle $v \in \mathbb{R}^L_+$ consumed at a node ξ is transformed into a bundle $Y_\mu v$ at each $\mu \in \xi^+$, where Y_μ is a $(L \times L)$-matrix with non-negative entries. The price of commodity $l \in L$ at node $\xi \in D$ is denoted by $p_{\xi,l}$.

Financial instruments. At each non-terminal node ξ there is a finite and ordered set $J(\xi)$ of collateralized credit contracts. Promises associated with $j \in J(\xi)$ are pooled into a pass-through security that distributes borrowers' payments. The pass-through security associated with a credit contract j is identified with the same index, and we assume that its price coincides with the price of the underline credit contract, denoted by $q_{\xi,j}$. Additionally, each security $j \in J(\xi_0)$ could be renegotiated at a price $q_{\xi,j}$ at any intermediate node $\xi \in D_1$.

The space of commodity and financial prices, denoted by $\mathcal{P}$, is given by vectors (p, q) such that $p \in (\mathbb{R}^L_+ \setminus \{0\})^D$ and $q \in \mathbb{R}_+^{(D\setminus D_2)\times J(\xi_0)} \times \prod_{\xi\in D_1} \mathbb{R}_+^{J(\xi)}$.[6]

Financial trading rules. The seller of one unit of credit contract $j \in J(\xi)$ receives an amount of resources $q_{\xi,j}$, is burdened to pledge a physical collateral $C_{\xi,j} \in \mathbb{R}^L_+ \setminus \{0\}$, and promises to pay a coupon $A_{\mu,j}(p, q)$ at each successor node μ of ξ. It is assumed that borrowers hold and consume collateral guarantees. Furthermore, each credit line incorporates a prepayment rule, which specifies the payment needed to reduce the amount of debt before terminal nodes. More precisely, borrowers of $j \in J(\xi_0)$ can reduce at an intermediate node ξ their short-positions in one unit by paying an amount of resources $B_{\xi,j}(p, q)$.

It follows that at each intermediate node $\xi \in D_1$ the closing cost of one unit of debt $j \in J(\xi_0)$ is given by the minimum between the prepayment cost and the collateral value, i.e., $R_{\xi,j}(p, q) := \min\{B_{\xi,j}(p, q),\ p_\xi C_{\xi,j}\}$, where $C_{\xi,j} := Y_\xi C_{\xi_0,j}$. In addition, since the only enforcement in case of default is the seizure of the associated collateral, at each terminal node $\xi \in D_2$ the closing cost of one unit of debt j is given by $R_{\xi,j}(p, q) := \min\{A_{\xi,j}(p, q), p_\xi Y_\xi C_{\xi^-,j}\}$.

At intermediate nodes, heterogeneous debt payment decisions may be observed as a consequence of credit shrinkages: some agents may pay, while others could prepay or default on their promises. However, all borrowers adopt the same decision at terminal nodes: they honor promises if and only if the coupon is less than or equal to the value of the associated collateral guarantees.

Given a non-terminal node ξ, buyers of one unit of security j pay $q_{\xi,j}$, which entitles them to obtain a payment $N_{\mu,j}$ at each successor node μ of ξ. Unitary payments are endogenously determined in equilibrium such that, node by node, resources distributed to lenders of security j match borrowers' deliveries. The space of security payments is given by $\mathcal{N} = \mathbb{R}_+^{D^+}$, where $D^+ = \{(\mu, j) : \exists\xi$

[4] According to CoreLogic data: http://www.corelogic.com.

[5] The US non-recourse states are Alaska, Arizona, California, Connecticut, Idaho, Minnesota, North Carolina, North Dakota, Oregon, Texas, Utah, and Washington. Even in these states default entails additional costs such as taxes (Form 1099-A) and negative credit ratings. However, our model identifies a novel channel that rationalizes the existence of underwater mortgages without any additional enforcement mechanism.

[6] Our definition of $\mathcal{P}$ does not allow the vector of commodity prices to be zero. However, this definition entails no loss of generality as we later assume the strict monotonicity of preferences.