



Consumer surplus analysis under uncertainty: A general equilibrium perspective

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ABSTRACT

This paper derives an exact form of partial equilibrium efficiency measure under uncertainty which is consistent with expected utility maximization in a general equilibrium situation with ex-post spot markets for many goods and asset markets which are in general incomplete.

We consider that the good under consideration tends to be negligibly small compared to the entire set of commodity characteristics which is assumed to be a continuum, and look into the limit property of preferences over state-contingent consumption of the good and state-contingent income transfer associated to it. We show that the limit preference exhibits risk neutrality, not only that it exhibits no income effect, meaning that the two conditions are tied together. We also show that the marginal rate of substitution between extra income transfers at different states of the world converges to the ratio between the Lagrange multipliers associated to those states. When the asset markets are complete such ratios are equalized between consumers, but it is not the case in general when the asset markets are incomplete. This means that using the aggregate expected consumer surplus as the welfare measure will be in general inconsistent with individuals' expected utility maximization in the general equilibrium environment or with ex-ante Pareto efficiency.

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1. Introduction

1.1. Motivation

The role of (aggregate) expected consumer surplus as an efficiency measure is prominent in many fields which adopt the partial equilibrium framework under uncertainty, such as mechanism design, industrial organization, environment, health, agriculture and others.

To recall the definition, suppose there are S states of the world, let (x_1, \dots, x_S) denote the vector of an individual's state-contingent consumptions of the commodity under consideration, where x_s denotes his consumption of the good at state $s = 1, \dots, S$. Let (a_1, \dots, a_S) denote the vector of state-contingent income transfers to him, where a_s denotes ex-post transfer at state $s = 1, \dots, S$. Then the expected consumer surplus for the given individual takes the form

$$\sum_{s=1}^S (v(x_s) + a_s) \pi_s$$

or its arbitrary monotone transformation, where (π_1, \dots, π_S) denotes the probability vector.

The usual textbook/classroom remark for this is that it relies on two assumptions: (1) no income effect in the sense that the marginal rate of substitution of income transfer at state s and consumption of the good contingent on state s is independent of a_s ; (2) risk neutrality in the sense that evaluation of uncertain prospects in the above form depends only on the expectation of consumer surplus $\sum_{s=1}^S (v(x_s) + a_s) \pi_s$ without any further adjustment, or in other words the marginal rates of substitution between income transfers at different states are constant.

We would point out one more assumption which is critical in aggregation: (3) values of income transfers are equal across states of the world, and also equal across individuals; or at least, the way how the values of income transfers differ across states is the same across individuals.

It remains unclear, however, if and how the three assumptions can be said precisely and consistently in the words of the general equilibrium theory.

In complete market settings, the assumption of no income effect has been given a general equilibrium theoretic characterization. Vives (1987) considers an increasing sequence of sets of commodities, and shows that income effect on each single commodity vanishes as the number of commodity and income tend to infinity at the same rate (see also Hayashi, 2008 for some follow-up to it). Hayashi (2013) instead starts with presenting the whole set of

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commodities as a continuum and subdivide it into many pieces so that each piece tends to be arbitrarily small, and shows that willingness to pay for a commodity is established in the limit as a density notion and exhibits no income effect.

The key here is to make a distinction between the pool of income held by the consumer beforehand (let us call it *background income* to emphasize the distinction) and relative change from it by means of transfer. The small income effect result therefore states that if the background income is sufficiently large compared to the given commodity or the commodity is sufficiently small compared to the background income then the effect of income transfer on the demand for it is negligible.

For the case of uncertainty, the above arguments are readily extended to Arrow–Debreu markets (Arrow, 0000; Debreu, 1959) in an essentially deterministic manner, in which commodities are differentiated by date-events at which they are delivered as well as by their material characteristics. Let T be the set of material characteristics of commodities and S be the set of states of the world. Then we can consider an extended set of commodity characteristics $T \times S$ and take let us say (t, s) to be the object of partial equilibrium analysis, in which we look at how consumers are willing to substitute between commodity (t, s) and income transfer to be spent over the *rest* of commodities $(T \times S) \setminus \{(t, s)\}$.

However, this way of extension presumes that value of income is uniform across all date-events, and it does not answer our question about if and why values of income transfers are equalized across states of the world and also equalized across individuals.

This necessitates to look into a Hicksian-type aggregation problem in a general equilibrium setting under uncertainty, such that:

1. there are “many” commodities in the spot markets to be opened at each state of the world, and the commodity under consideration is a “negligibly small” one;
2. there are asset markets in which individuals allocate their incomes across states, either completely or incompletely.

The first element captures the above-noted issue on the no-income-effect assumption. It reconfirms the classic “excuse” since Marshall (1920), saying that when the commodity is negligibly small compared to the entire set of commodities one can ignore income effect on it.

The second element captures the fact that consumers’ risk attitudes in partial equilibrium and the values of income transfers at different states are determined *endogenously* by how they take positions in the asset markets. This fact is captured better by adopting the asset market model due to Radner (1968), rather than the Arrow–Debreu model (Arrow, 0000; Debreu, 1959), which is the case particularly when the asset markets are incomplete.

1.2. Outline of results

In order to deal with the above issues, we work on the problem of Hicksian aggregation under uncertainty in the setting due to Radner (1968). We derive consumer’s indirect preference over state-contingent consumptions of a given material good and state-contingent transfers of income that is to be spent on the other goods in the spot markets at each state. We take the set of material characteristics of commodities as a continuum, and take the given good as an element of its finite partition. We consider a limit in the sense that the partition becomes arbitrarily finer and the good tends to be arbitrarily small, while the magnitude of income transfers is adjusted to the smallness of the good and tends to be small as well. Given a finite partition, the argument falls in the standard demand theory in the literature of general equilibrium with incomplete markets (GEI) such as Geanakoplos and Polemarchakis (1986) and Magill and Quinzii (2002). The current work may be viewed as a contribution to the demand theory in the GEI setting with infinitely many commodities in the spot markets, as we establish the existence and uniqueness of the limit.

We look into the limit property of preference over state-contingent consumption of the good and state-contingent income transfer associated to it. We show that the limit preference is risk-neutral, not only that it exhibits no income effect, meaning that the two conditions are tied together. We also show that the marginal rate of substitution between extra income transfers at different states of the world converges to the ratio between the Lagrange multipliers associated to those states. When the asset markets are complete such ratios are equalized between consumers, but it is not the case in general when the asset markets are incomplete. This means that using the aggregate expected consumer surplus as the welfare measure will be in general inconsistent with individuals’ expected utility maximization in the general equilibrium environment or with ex-ante Pareto efficiency.

1.3. Related literature

Let us conclude the introduction by discussing the relation between the present paper and papers on evaluating uncertain price–income pairs or uncertain incomes.

It is known that the expected consumer surplus criterion concludes price instability is good (see for example Waugh, 1944 and Massell, 1969). Consider that inverse demand curve is linear or that it is locally approximated linearly. Say it is $p(x) = 1 - x$. Then consumer surplus given price p is $(1 - p)^2/2$, which is convex in p and implies p being more risky is good. This point has motivated careful examination of preference over price uncertainty.

Rogerson (1980) considers preference over probability distribution of price–income pairs in ex-post spot markets, basically represented in the expected utility form $U(F) = \int V(p, m)dF(p, m)$, in which the von-Neumann/Morgenstern index V defined over price vector p and income m is supposed to play the role of indirect utility function in the ex-post spot markets as well as to describe the consumer’s risk attitude toward price–income uncertainty.^{1,2} Rogerson shows that expected consumer surplus from a good represents the consumer’s preference over distributions of its price given the same income if and only if V is additively separable between the price and income. The result is sophisticated by Schlee (2008), who shows a stronger result that the equivalence is true even for approximate representation, and also that the aggregate expected consumer surplus is consistent with Kaldor criterion if and essentially only if V is linear in income. In a more recent paper, Schlee (2012) provides a more robust sufficient condition under which maximization of expected consumer surplus leads to Pareto efficiency.

In the above approach the asset markets and how consumers take positions there are taken to be an implicit fixed factor. The current paper is taken to be endogenously deriving consumers’ risk attitude over prices and incomes from their decisions in the asset market.

2. Hicksian aggregation under uncertainty

2.1. Consumption and price spaces

First we describe the consumption space and the associated price space in a deterministic setting, before introducing uncertainty, and introduce some relevant mathematical concepts.

Let $T = [0, 1]$ be the set of commodity characteristics, Σ be the family of Lebesgue measurable sets, and μ be the Lebesgue measure.

¹ In basically the same setting, Turnovsky et al. (1980) provide a sufficient condition under which price stabilization is good.

² Grant et al. (1992) provide characterizations of when preference over lotteries over consumption of many goods, which is taken to be the primitive, can be described as a preference over money lotteries.

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