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Optimal agglomerations in dynamic economics☆

William A. Brock[a,b], Anastasios Xepapadeas[c], Athanasios N. Yannacopoulos[d,*]

[a] Department of Economics, University of Wisconsin, United States
[b] Department of Economics, University of Missouri, United States
[c] Department of International and European Economic Studies, Athens University of Economics and Business, Greece
[d] Department of Statistics, Athens University of Economics and Business, Greece

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ABSTRACT

We study rational expectations equilibrium problems and social optimum problems in infinite horizon spatial economies in the context of a Ramsey type capital accumulation problem with geographical spillovers. We identify sufficient local and global conditions for the emergence (or not) of optimal agglomeration, using techniques from monotone operator theory and spectral theory in infinite dimensional Hilbert spaces. We show that agglomerations may emerge, with any type of returns to scale (increasing or decreasing) and with the marginal productivity of private capital increasing or decreasing with respect to the spatial externality. This is a fairly general result indicating the importance of the network structure of the spatial externality relative to the properties of the aggregate production function. Our analytical methods can be used to systematically study optimal potential agglomeration and clustering in dynamic economics.

1. Introduction

This paper shows how monotone operator theory can be used to study rational expectations equilibrium (REE) problems and social optimum (SO) problems in infinite horizon, infinite dimensional spatial economies. Our analysis is applied to an illustrative infinite horizon, infinite dimensional spatial Ramsey type capital accumulation problem where borrowing and lending on world capital markets at a rate of interest equal to the rate of discount on subjective utility are the same; quadratic adjustment costs penalize rapid movements of capital; and geographical spillovers stemming from capital accumulation across location generate a positive externality. We locate sufficient conditions on primitives that may cause potential agglomerations and spatial clustering to form or not form for both problems. Furthermore, we show how the spectral theory of compact operators allows decomposition of the infinite dimensional problem into a countable collection of tractable finite dimensional problems. Using this technique we provide explicit local stability criteria for the linearized system.

Related literature in terms of new economic geography includes work by Krugman (1996), Fujita et al. (2001), Lucas (2001), Quah (2002), Desmet and Rossi-Hansberg (2007), Ioannides and Overman (2007), Lucas and Rossi-Hansberg (2002), and others. However, to our knowledge, no one has yet provided a concise framework in which the combination of monotone operator theory, the theory of compact operators, and the decomposition techniques we develop here can be applied to infinite horizon, infinite dimensional spatial economies to study endogenous agglomeration (or non-agglomeration) for rational expectations equilibrium and the social optimum in terms of local and global analysis for Ramsey type growth models as we do here.

There is a small but growing literature which studies optimal dynamic social welfare, e.g., an analog of our SO problem in terms of Solow, Ramsey and AK type models with a trade balance where capital is mobile across space and growth occurs. In these models capital movement is modeled by local diffusion and spatial dynamics are governed by a parabolic partial differential equation (e.g., Camacho and Zou (2004), Camacho et al. (2008), Boucekkine et al. (2009), Brito (2011) and Boucekkine et al. (2013a,b)). These models also study the long-run structure of the spatial distribution of the stock of capital and the results suggest convergence to a spatially homogeneous steady state.

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* Corresponding author.
E-mail addresses: wbrock@ssc.wisc.edu (W.A. Brock), xepapad@aueb.gr (A. Xepapadeas), ayannaco@aueb.gr (A.N. Yannacopoulos).

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In contrast and complementary to this literature, the present paper studies both the SO and REE problems under non-local spatial effects which can be regarded as a spatial Romer–Lucas type of externality and derives conditions, based on the network structure of the spatial externality, for potential agglomeration emergence.

It is known that agglomerations and spatial clusters may appear with localized positive spatial spillovers when there are increasing returns. In this case the increasing returns activity concentrates to one location (e.g. Grossman and Helpman (1991)). Actually increasing returns underlie the generation of centripetal forces that favor cumulative causation and thus spatial clustering (e.g., Nocco (2005)). In our model the production technology exhibits diminishing marginal productivity with respect to private capital for any fixed value of the spatial externality and diminishing returns with respect to the spatial externality for fixed levels of private capital, although increasing social returns, in the sense of Romer (1986), are possible.

In the context of a Ramsey type growth model with the above production technology, our contribution consists of providing conditions for the emergence or not of agglomerations which does not depend on the structure of the aggregate production technology, that is increasing versus decreasing returns, but rather on the network structure of the spatial externality. We show that this structure is important since it may induce endogenous agglomeration with decreasing returns to scale and without exogenous agglomeration drivers such as boundary conditions or location advantages. Thus, combining global analysis based on monotone operator theory with local analysis based on spectral theory, we obtain valuable insights regarding the endogenous emergence (or not) of optimal agglomerations at an REE and the SO of dynamic economic systems modeled by a Ramsey type growth model.

In terms of global analysis we show that long run agglomerations do not emerge at the SO, where full internalization of the spatial externality by the social planner occurs, and the system converges to a unique spatially homogeneous (or flat) steady state. This result suggests that we have identified a class of problems for which the usual value loss methods of proving turnpike theorems, which depend upon a small enough discount rate on future payoffs (e.g., McKenzie (1976), Araujo and Scheinkman (1977) and Bewley (1982)), can be replaced by monotone operator methods of getting asymptotic convergence results in settings where the state space is infinite dimensional and where the results are not so dependent upon a small discount rate on future payoffs. Furthermore we treat cases in which there are externalities and solutions are not necessarily Pareto optimal. For an REE, where incomplete internalization of the spatial externality by optimizing agents occurs, we show that if the network structure of the spatial interactions is such that the operator characterizing spatial spillovers is monotonic then the system also converges to a unique spatially homogeneous steady state, in general not the same as the SO steady state. On the other hand, if monotonicity does not hold, then multiple steady states are possible in an REE. In this case only one steady state will be spatially homogeneous and therefore the long-run REE could be characterized by agglomerations.

In terms of local analysis we use spectral theory to derive conditions for instability to spatial perturbations of a spatial homogeneous steady state which may lead to agglomerations. We show that having potential agglomerations at an REE does not require increasing returns to scale or the marginal productivity of private capital to be positively related to the spatial externality. On the contrary, given the structure of the spatial externality, spatial agglomeration may emerge with decreasing returns to scale and diminishing marginal productivity of private capital with respect to the spatial externality. This result indicates the importance of the network structure relative to the properties of the aggregate production function, suggesting that spatial agglomeration may emerge as the outcome of an REE when returns to scale are decreasing and nonlocal spatial spillovers occur. It is also important to note that long-run agglomerations do not emerge at the SO where the spatial externality is fully internalized and returns to scale are decreasing. Thus our result establishes a potential divergence, in terms of the spatial distribution of the stock of capital, between equilibrium outcomes and socially optimal outcomes. This divergence may appear even under decreasing returns to scale.[1] Numerical simulations confirm the results of our local analysis.

We would also like to note that there is a large literature in mathematical biology (e.g., Murray (2003)) that studies spatial agglomeration problems in infinite dimensional spaces. However, as far as we know, none of this literature deals with optimization problems as we do here. There are many differences between the "backward-looking" dynamics in mathematical biology problems and other natural science problems, and the "forward-looking" dynamics of economic problems. It is not just a simple adaptation of dynamical systems techniques to two-point boundary value problems similar to the familiar phase diagrams in textbook analysis of Ramsey type optimal growth problems and Ramsey type rational expectations problems in finite dimensional spaces. For example, our development of techniques from operator theory mentioned above allows us to locate sufficient conditions on primitives for all potential agglomerations to be removed in infinite horizon optimization problems. Furthermore, and contrary to the spirit of the Turing instability, which provides local results for non optimizing linearized dynamical systems, we obtain global results valid for the fully nonlinear optimized dynamical system.

The paper is organized as follows: Section 2 introduces the model and Section 3 characterizes equilibria with spatial spillovers. Sections 4 and 5 provide global and local analysis for the emergence (or not) of optimal potential agglomerations while Section 6 presents a detailed analytic and numerical example. Section 7 discusses intuition, shows how our methods can be used to study generalizations to spatial domains of similarly structured economic problems – in this case the well known investment problem of the firm with adjustment costs – and outlines other ways in which the present paper can be extended. So as not to disrupt the flow of the presentation, all proofs are contained in the Appendix.

2. Geographical spillovers in forward-looking optimizing economies

Consider a spatial economy occupying a bounded domain $\mathcal{O} \subset \mathbb{R}^d$. It is worth noting that space may be considered as either geographical (physical) space or as economic space (space of attributes related to economic quantities of interest). Without loss of generality we may assume $d = 1$.

Capital stock is assumed to be a scalar quantity that evolves in time and depends on the particular point z of the domain $\mathcal{O}$ under consideration. Thus capital is described as a function of time t and space z, i.e. $x : I \times \mathcal{O} \to \mathbb{R}$ where $I = (0, T)$ is the time interval over which the temporal evolution of the phenomenon takes place. We assume an infinite horizon model, i.e. $I = \mathbb{R}_+$, and denote the capital stock at point $z \in \mathcal{O}$ at time t by $x(t, z)$. The spatial behavior of x is modeled by assuming that the functions $x(t, \cdot)$ belong for

[1] It would be interesting to model the effect of introducing spillover externalities like ours as well as introducing growth and local effects like Boucekkine et al. (2013a,b) and comparing the solutions for SO and RE as we do in our paper. This is, however, beyond the scope of the current paper.