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Equality of opportunity and the alignment of incentives[☆]

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ABSTRACT

Under what conditions are the incentives of the members of society more aligned? We address this question in a setup in which individuals choose a policy without knowing who will benefit and who will be hurt by each policy. Our central result identifies a sufficient condition for a measure of disagreement in society, which has been linked to the equilibrium level of rent-seeking, to weakly decrease. This sufficient condition captures increasing equality of opportunity in a specific sense.

1. Introduction

There is a nice fable about an eagle, a crab, and a pike trying to pull a cart. Because they pull in different directions, they do not get very far. This simple fable illustrates a profound point: that a society whose members have vastly disparate interests will have difficulty making progress, say, because much effort is expended on unproductive rent-seeking activities. This raises the following question: under what conditions are the incentives of the members of society more aligned?

The current paper addresses this question in a setup with the following key features: (i) the members of society must choose an alternative (or policy); (ii) the desirability of a given alternative for each member depends on the position in which she (or her offspring) ends up; (iii) at the time of choosing an alternative, each member of society does not know for sure in which position she will end up; and (iv) different individuals may face different probabilities of ending up in each position. This setup represents a kind of generalized veil of ignorance, one that does not necessarily treat all individuals symmetrically (because of (iv)). As a result, individuals may disagree about the best alternative and may engage in rent-seeking activities as they try to influence society's choice of alternative.

These features are likely to be present in many real-world situations of social choice, especially when the choice is about some general-level policy. For example, when deciding whether to adopt general-level free-trade policies (e.g., by joining the World Trade Organization), individuals may not know who will end up in the position of a consumer who benefits from these policies and who will end up in the position of a domestic producer who is hurt by these policies.[1] Nevertheless, some members of society may be more likely to end up as consumers while other members may be more likely to end up as domestic producers. As a result, the former/latter group of individuals may spend resources (say, by lobbying politicians) in order to support/oppose free-trade policies.

Our formal notion of what it means for incentives to be more aligned is based on a direct measure of disagreement in society that is likely to be a good proxy for rent-seeking activities. In particular, this measure has been linked by Esteban and Ray (2011) to the equilibrium amount of resources spent on rent-seeking in a game-theoretic model of rent-seeking.[2,3]

The main result of the paper identifies a sufficient condition for incentives to become more aligned in the sense that the measure of disagreement weakly decreases. This condition captures the idea of greater equality of opportunity and is about each member of society facing a probability distribution over positions that is, in a specific sense, closer to the uniform distribution. This condition,

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[1] If the policies are more specific, say, about whether to lower tariffs in the steel industry, individuals may know with relative certainty which policies will benefit them, so that point (iii) above would not apply.

[2] As explained later, the link is not theoretically exact, but holds up very well in numerical simulations.

[3] Esteban and Ray use the term "conflict" rather than "rent-seeking". The terminology is unimportant. The point is that resources are being wasted.

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called position-uniformly greater equality of opportunity, is quite strong. However, other notions of greater equality of opportunity or social mobility from the existing literature are not sufficient for our theoretical result.

The paper is organized as follows. Section 2 describes the general framework for our analysis. Section 3 presents the measure of disagreement. Section 4 introduces our notion of position-uniformly greater equality of opportunity and provides the main result. Section 5 provides a justification for the measure of disagreement by linking it to the equilibrium amount of resources spent on rent-seeking. Section 6 provides a literature review. Section 7 concludes.

2. General framework

We start with the following definition, which provides our formal framework.

Definition 1. A situation of social choice behind a generalized veil of ignorance is a tuple $\langle I, J, \mathbf{P}, X, \succeq \rangle$, where:

- $I = \{i_1, \ldots, i_n\}$ is a set of individuals ($n \geq 2$);
- $J = \{j_1, \ldots, j_n\}$ is a set of positions;
- $\mathbf{P}$ is an $n \times n$ bistochastic matrix,[4] in which the kth-row, lth-column entry, $\mathbf{P}_{kl}$, shows the probability with which individual i_k ends up in position j_l;
- X is a set of feasible alternatives;
- $\succeq$ is each individual's preference relation over simple probability distributions[5] over $X \times J \times \mathbb{R}$ (where $\mathbb{R}$ stands for amounts of "money"), which satisfies transitivity, completeness, the independence axiom, and the Archimedean axiom.[6]

Note that, given the assumptions on $\succeq$, this preference can be represented by the expectation of a (von Neumann–Morgenstern) utility function $\tilde{u} : X \times J \times \mathbb{R} \to \mathbb{R}$. We further assume that the last argument of $\tilde{u}$ (money) plays the role of a numeraire, i.e., we assume $\tilde{u}(x, j, m) = u(x, j) + m$.

In a situation of social choice behind a generalized veil of ignorance, n individuals are randomly assigned to n positions (one individual per position). The kth row in $\mathbf{P}$, $\mathbf{P}_k$, shows the probabilities with which individual i_k ends up in positions $j_1, \ldots, j_n$. Further, society has to choose an alternative $x \in X$. An individual's utility from a given alternative depends on the position in which she ends up. Each individual i_k evaluates an alternative $x \in X$ (holding the numeraire fixed) according to the expectation over $j \in J$ of $u(x, j)$, where this expectation is computed based on $\mathbf{P}_k$. Individuals are behind a veil of ignorance because, when deciding on the alternative $x \in X$, they do not know in which positions they will end up. The veil is "generalized" because, unlike in the case of the original veil of ignorance, individuals need not face identical uniform distributions over positions (the rows of $\mathbf{P}$ can differ).

In this setup, individuals facing different probability distributions over positions can disagree about the optimal policy.[7] As a result, individuals may try to influence society's choice. This can be done by spending numeraire on rent-seeking activities, such as lobbying, demonstrating, engaging in media campaigns, etc.[8]

The following examples may elucidate matters.

1. Some positions in J correspond to consumers and other positions correspond to domestic producers. The set of feasible alternatives, X, consists of different possible levels of barriers to international trade. Consumers/domestic producers prefer free-trade/protectionist policies.[9]
2. Some positions in J correspond to consumers and other positions correspond to monopolists. The set of feasible alternatives, X, consists of different possible kinds of anti-trust legislation. Consumers/monopolists prefer more/less stringent anti-trust laws.
3. Some positions in J correspond to ordinary citizens and other positions correspond to polluting producers. The set of feasible alternatives, X, consists of different possible kinds of environmental protection policies. Ordinary citizens/polluting producers prefer more/less stringent environmental protection.
4. Some positions in J are associated with low-income levels and other positions are associated with high-income levels. The set of feasible alternatives, X, consists of different possible kinds of redistribution systems (more or less progressive taxes, more or less generous social safety-net programs). Individuals in low-income/high-income positions prefer more/less redistributive systems.
5. The individuals in I are politicians before an election that will put some of them in executive authority. Some positions in J correspond to the winners of the election and other positions correspond to the losers. The set of feasible alternatives, X, consists of different sets of rules governing the powers of the executive. (These rules can be constitutional rules, procedural rules, etc.) Winners/losers prefer rules that grant more/less power to the executive.

3. A measure of disagreement

In this section, we present a measure of disagreement in society. Assume $\mathbf{P}$, X, and u are such that each individual i_k has a unique optimal alternative (holding the numeraire fixed). That is, assume that, for each $k \in \{1, \ldots, n\}$, there exists an alternative, $x(\mathbf{P}_k) \in X$, that uniquely solves $\max_{x \in X} \mathbf{P}_k \begin{pmatrix} u(x,1) \\ \vdots \\ u(x,n) \end{pmatrix}$.[10] Let $\mathbf{u}(\mathbf{P}_k) = \begin{pmatrix} u(x(\mathbf{P}_k),1) \\ \vdots \\ u(x(\mathbf{P}_k),n) \end{pmatrix}$, i.e., $\mathbf{u}(\mathbf{P}_k)$ denotes the vector of utilities obtained by an individual in each position if i_k's optimal alternative is chosen.

Define $\delta(\mathbf{P}_k, \mathbf{P}_l) = \mathbf{P}_k \mathbf{u}(\mathbf{P}_k) - \mathbf{P}_k \mathbf{u}(\mathbf{P}_l)$. $\delta(\mathbf{P}_k, \mathbf{P}_l)$ shows the expected utility loss (in units of the numeraire) that individual i_k would suffer if individual i_l's optimal alternative is chosen rather

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4 A bistochastic matrix is a square matrix in which (i) all entries are nonnegative real numbers and (ii) each row and column sums to 1.

5 That is, probability distributions with finite support.

6 These are the standard axioms in expected utility theory. See Kreps (1988, pages 7, 8, and 44).

7 Note that we are assuming that all individuals share the same preference $\succeq$ over lotteries over $X \times J \times \mathbb{R}$ and, hence, the same utility function u. Differences between individuals arise solely because individuals face different probability distributions over positions.

8 Each individual is implicitly assumed to start out with some endowment of the numeraire. To avoid boundary issues, we allow an individual to end up with negative quantities of the numeraire (the last argument of $\tilde{u}$ can be any number). Given this, the precise endowments are irrelevant and, hence, are omitted from the definition above.

9 Letting (x, j, m) denote the degenerate lottery that leads to alternative $x \in X$, position $j \in J$, and monetary outcome m with probability 1, the formal way to state the last sentence is: (free trade policy, consumer, m) $\succeq$ (protectionist policy, consumer, m) and (protectionist policy, monopolist, m) $\succeq$ (free trade policy, monopolist, m). A similar remark applies to the following examples as well.

10 The existence of an optimal alternative is assured either if X is finite or if X is a compact topological space and $u(x, j)$ is continuous in x for $j \in \{1, \ldots, n\}$. Geometrically, uniqueness is equivalent to the requirement that, for each k, the "highest" hyperplane in $\mathbb{R}^n$ with normal vector $\mathbf{P}_k$ touching the set $U(X) = \{\mathbf{v} \in \mathbb{R}^n | \mathbf{v} = (u(x, j_1), \ldots, u(x, j_n))$ for some $x \in X\}$ touches this set at a single point.