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Dynamical response properties of a canonical model for type-I membranes

B. Naundorf*, T. Geisel, F. Wolf

Department of Nonlinear Dynamics, Max-Planck-Institut für Strömungsforschung and Fakultät für Physik, Universität Göttingen, 37073 Göttingen, Germany

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Abstract

We study a canonical model of type-I membranes subject to correlated fluctuating input currents. We present a semi-analytical approach for calculating the response of this neuron model to time dependent inputs both in the input current and the noise amplitude using a novel sparse matrix representation of the systems Fokker–Planck operator. It turns out, that the maximum stimulation frequency which can be transmitted through this model neuron is approximately given by the stationary firing rate. Our results agree well with the behavior of a conductance-based model-neuron but are in qualitative disagreement with key response properties of leaky integrate-and-fire neurons.

Keywords: Population of spiking neurons; Dynamics; Spike initiation; Integrate-and-fire model

1. Introduction

Cortical neurons in vivo are subject to an immense synaptic bombardment which leads to rapid fluctuations in their membrane potential (MP) [4]. For analytical, as well numerical studies it is crucial to identify simple, yet biophysical realistic neuron models, which reproduce the dynamical behavior of real neurons under such conditions. Here we present the stationary and dynamical response properties of a

*Corresponding author. Tel.: +49 551 5176431.
E-mail address: bjoern@chaos.gwdg.de (B. Naundorf).

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canonical model of membranes exhibiting type-I excitability in a biologically realistic regime. For a temporally correlated input current we expand the Fokker–Planck operator into a suitable basis set, for which the matrix representation is sparse. Subsequent diagonalization of the resulting Fokker–Planck matrix allows for a very fast and efficient computation of the firing rate, as well as the stationary density of the MP. In the second part we study dynamical responses based on linear response theory. We show that the speed of responses is closely connected to the spectral properties of the Fokker–Planck operator and identify a cut-off frequency, which is approximately given by the stationary firing rate of the model neuron. Above this cut-off frequency the response amplitude is strongly damped. We show that this behavior is shared by a conductance based model neuron and is in qualitative disagreement with results reported on the leaky integrate-and-fire (LIF) model. Our study corroborates and extends recent reports [5,8], indicating that the LIF model is incapable of mimicking the dynamical response properties of conductance based model neurons. In addition our new method provides efficient computational tools for the analysis of dynamical neuronal responses.

2. Model

In our study we use a canonical model of type-I membranes. This model undergoes a saddle node bifurcation when brought to repetitive firing. Close to the bifurcation it can be shown that the complex dynamics of a multidimensional conductance based model can be described by its normal form [12]. For a saddle-node bifurcation this is given by $C\dot{V} = A(V - V^*)^2 + (I - I_c)$, which is a dynamical equation for the MP, V of the neuron. The rheobase of the neuron is denoted by I_c, the constants A and V^* can be deduced from the multidimensional conductance based model (see Fig. 1). It is convenient to introduce dimensionless quantities $\bar{V} = \tau A(V - V^*)/C$ and $\bar{I} = A(I - I_c)\tau^2/C^2$, where we have introduced the effective time constant τ. The rescaled dynamics is then given by

$$\tau\dot{\bar{V}} = \bar{V}^2 + I. \tag{1}$$

For an input current which is larger than the rheobase, i.e. $I > 0$, the MP has a finite "blow-up" time, which means that it needs a finite time to get from $-\infty$ to $+\infty$, where the latter is identified with the emission of a spike. The normal form Eq. (1) can be formally transformed into a phase oscillator, the θ-neuron [7], by substituting $\bar{V} = \tan(\theta/2)$

$$\tau\dot{\theta} = (1 - \cos\theta) + I(1 + \cos\theta) \tag{2}$$

with the angle variable θ in the interval $(-\pi, \pi]$. Here, a spike is emitted each time θ reaches the value π. In the following the input I will be decomposed into two parts, $I = I_0 + \sigma z(t)$, a mean input current I_0 and a stationary fluctuating part $\sigma z(t)$ with $\langle z(t)\rangle = 0$ and the correlation function $\langle z(t)z(t')\rangle \propto \exp(-(t - t')/\tau_c)$.

The parameters τ, I_0 and σ have been chosen to reflect the conditions of a cortical neuron in vivo. A spike takes about 1 ms, to reflect this we choose $\tau = 0.25$ ms.