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Stochasticity in localized synfire chain

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Abstract

We report on stochastic evolutions of firing states through feedforward neural networks with Mexican-Hat-type connectivity. The variance in connectivity, which depends on the pre-synaptic neuron, generates a common noisy input to post-synaptic neurons. We develop a theory to describe the stochastic evolution of the localized synfire chain driven by a common noisy input. The development of a firing state through neural layers does not converge to a certain fixed point but keeps on fluctuating. Stationary firing states except for a non-firing state are lost, but an almost stationary distribution of firing state is observed.

Keywords: Localized synfire chain; Stochastic evolution equations; Order parameters; Fourier modes

1. Introduction

A homogeneous feedforward network has been proposed as a simple model of transmitting a synchronous activity and has been intensively studied theoretically with several models of spiking neuron [1,5,6,8]. Stochastic evolution of propagating activity through a feedforward network has been studied recently in a simple binary neuron model [2]. The stochasticity is generated by common noise which comes from some properties of network structure, e.g., the sparseness of synaptic connections, or variance of connection efficacy. The pre-synaptic-dependent variance in connection

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efficacy produces common input to post-synaptic neurons. When the efficacy fluctuates from trial-to-trial, they result in fluctuating common noise.

The activity in the homogeneous feedforward networks is uniform. The brain, however, is not homogeneous. A feedforward network with Mexican-Hat-type connectivity (FMH) has been studied more recently [7,9]. It demonstrates stable propagation of a localized activity, but the effect of common noise originating connectivity variance on a localized activity has not yet been studied.

The present paper examines the FMH to study stochastic evolution of a localized activity driven by a common noise. Our strategy is describing the evolution of the firing states through order parameter equations by using the cosine function to represent the Mexican-Hat type connectivity [4].

2. Model

The network model used in this paper is described as follows:

$$x_\theta^{l+1} = \Theta(h_\theta^{l+1}) = \Theta(\Sigma_{\theta'} J_{\theta\theta'}^l x_{\theta'}^l - h),$$
$$J_{\theta\theta'}^l = -\frac{J_0}{N} + \frac{J_2}{N}\cos(2(\theta-\theta')) + w_{\theta\theta'}^l + w_{\theta'}^l, \tag{1}$$

where $x_\theta^{l+1} = \{0, 1\}$ is the output of the neuron on the $(l+1)$th layer at position θ. h_θ^{l+1} is the internal state of the neuron. One neuron at θ', where $\theta' = \{-\frac{\pi}{2}, -\frac{\pi}{2}+\frac{\pi}{N}, -\frac{\pi}{2}+\frac{2\pi}{N}, \ldots, \frac{\pi}{2}-\frac{\pi}{N}\}$, is making a synapse on a next-layer neuron at θ with connectivity $J_{\theta\theta'}^l$. Here, h is a threshold. Θ is a step function. Each layer consists of N neurons with a periodic boundary. $J_{\theta\theta'}^l$ is described by a cosine function. J_0 is a parameter of homogeneous connectivity and J_2 is the amplitude of Mexican-Hat type connectivity. $w_{\theta\theta'}^l$ and $w_{\theta'}^l$ correspond to some fluctuations of EPSPs or IPSPs described as the Gaussian distribution; $w_{\theta\theta'}^l \sim \mathcal{N}(0, \Delta^2/N)$ and $w_{\theta'}^l \sim \mathcal{N}(0, \delta^2/N)$.$w_{\theta\theta'}^l$ means the variance of connection efficacy between a pair of pre and post-synaptic neurons, which is independent of each connection. Here we introduce $w_{\theta'}^l$, which are pre-synaptic-dependent fluctuations, meaning heterogeneity of pre-synaptic neurons. If $w_{\theta'}^l > 0$ and the neuron at position θ' fires, the probabilities of emitting a spike for all the post-synaptic neurons are increased. Therefore, $w_{\theta'}^l$ is a source of common noise and the correlations of each neural activities. We define common noise as the sum of all the firing neuron's heterogeneity $w_{\theta'}^l$. Common noise is literally common to all the post-synaptic neurons, which fluctuates from trial to trial.

3. Theory: evolution equations for the order parameters

This paper introduces three order parameters, r_0^l, r_{2c}^l, and r_{2s}^l as the 0th and 2nd coefficients of the Fourier transformation of the firing state at the lth layer. The