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NEUROCOMPUTING
Neurocomputing 64 (2005) 359–374
www.elsevier.com/locate/neucom

An improved neural network for convex quadratic optimization with application to real-time beamforming

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Received 21 April 2004
Communicated by T. Heskes
Available online 8 January 2005

Abstract

This paper develops an improved neural network to solve convex quadratic optimization problems with general linear constraints. Compared with the existing primal–dual neural network and dual neural network for solving such problems, the proposed neural network has a lower complexity for implementation. Unlike the Kennedy–Chua neural network, the proposed neural network can converge to an exact optimal solution. Analyzed results and illustrative examples show that the proposed neural network has a fast convergence to the optimal solution. Finally, the proposed neural network is effectively applied to real-time beamforming.

Keywords: Quadratic optimization; Recurrent neural network; Convergence analysis; Real-time beamforming

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0925-2312/$ - see front matter
doi:10.1016/j.neucom.2004.11.009

1. Introduction

Consider the following quadratic optimization problem

$$\begin{aligned} \text{minimize} \quad & \tfrac{1}{2}x^{\mathrm{T}}Qx + c^{\mathrm{T}}x \\ \text{subject to} \quad & Bx = b, \quad Ax \leqslant d, \\ & l \leqslant x \leqslant h, \end{aligned} \tag{1}$$

where $Q \in R^{n\times n}$ is a symmetric and positive definite matrix, $B \in R^{m\times n}$, $A \in R^{r\times n}$, $c, h, l \in R^n$, $b \in R^m$, and $d \in R^r$. It is well-known that quadratic optimization problems arise in a wide variety of scientific and engineering applications including regression analysis, image and signal processing, parameter estimation, filter design, robot control, etc. [1]. Many of them have time-varying nature and thus have to be solved in real time [6,16]. Because of the nature of digital computers, conventional numerical optimization techniques may not be effective for such real-time applications. Neural networks are composed of many massively connected neurons. The main advantage of the neural network approach to optimization is that the nature of the dynamic solution procedure is inherently parallel and distributed. Unlike other parallel algorithms, neural networks can be implemented physically in designated hardware such as application-specific integrated circuits, where optimization is carried out in a truly parallel and distributed manner. Because of the inherent nature of parallel and distributed information processing in neural networks, the convergence rate of the solution process is not decreasing as the size of the problem increases. Therefore, the neural network approach can solve optimization problems in running time at the orders of magnitude much faster than the most popular optimization algorithms executed on general-purpose digital computers [3]. Neural networks for optimization have received tremendous interest in recent years [2,5,7,10,12–15,17]. At present, there are several recurrent neural networks for solving quadratic optimization problems (1). Kennedy and Chua [7] presented a primal neural network. Because the network contains a finite penalty parameter, it converges an approximate solution only. To overcome the penalty parameter, we proposed a primal–dual neural network and a dual neural network [12,14]. The primal–dual neural network has a two-layer structure and the dual neural network requires computing an inverse matrix. Thus, both neural networks have a model complexity problem. Moreover, all existing neural networks for solving (1) cannot be guaranteed to have an exponential convergence to the optimal solution of (1). Thus, studying alternative neural networks with a low complexity and a fast convergence rate is of importance and significance.

The objective of this paper is to develop an improved neural network for solving (1) with a low complexity and fast convergence. The proposed neural network has one-layer structure without the need of computing an inverse matrix. The beamforming processor is used to generate an optimal set of beams to track the mobiles within the coverage area of the base-station. The implemented algorithms have to rapidly enhance the desired signal and suppress noise and interference at the