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The calibration of CES production functions ☆

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ABSTRACT

The CES production function is increasingly prominent in macroeconomics and growth economics. This paper distinguishes between different uses of "normalized" CES functions, an approach that has become popular in the literature. The results of Klump and La Grandville (2000) provide a simple way to calibrate the parameters of the CES production function when the necessary data are available. But some of the other applications of normalized CES production functions are problematic, especially when the approach is said to isolate the theoretical effects of varying the elasticity of substitution.

1. Introduction

In recent years, the CES production technology has gained much greater prominence in growth economics and macroeconomics. It is the most popular alternative to (and generalization of) the Cobb-Douglas technology, and can be used to address a wider range of issues than Cobb-Douglas. At the same time, it is not always straightforward to justify particular choices for the CES technology parameters, or to examine their implications. Klump and La Grandville (2000) drew attention to this problem, and outlined a procedure for explicitly "normalizing" the production function. Their approach has become popular in the literature, but has sometimes been misused. This paper will distinguish between some of the possible uses of explicit normalization, and seek to clarify when it is useful, and when it may be misleading.

There are several reasons for the increasing popularity of the CES technology. These include the observed time-series and cross-section variation in factor shares in advanced economies, and the tendency for empirical studies using single-country data or microeconomic data to estimate the elasticity of substitution between capital and labor to be well below unity.[1] Some cross-country studies reject a unitary elasticity of substitution (Duffy and Papageorgiou, 2000). In growth models, technologies more flexible than Cobb-Douglas are needed for the consideration of varying factor shares, factor-biased technical change, and appropriate technology (for example, Acemoglu, 2003; Caselli, 2005 and Caselli and Coleman, 2006). A non-unitary elasticity also has implications for fiscal policy, as in the Backus et al. (2008) study of the cross-country relationship between

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[1] On factor shares, see Blanchard (1997), Bentolila and Saint-Paul (2006), and Aiyar and Dalgaard (2009); on estimates see, for example, Antràs (2004); and for surveys, see Chirinko (2008); Klump et al. (2008); Klump et al. (2011).

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capital-output ratios and corporate tax rates. More widely, theorists often consider the implications of variation in the elasticity of substitution, and it has been argued that the CES technology deserves much greater prominence in short-run macroeconomics.[2] Another sign of the renewed interest in the CES technology is that the *Journal of Macroeconomics* devoted a special issue to this production function in June 2008.

Although the CES production technology seems relatively straightforward, its mathematical simplicity can be deceptive. La Grandville (1989, 2009), Klump and La Grandville (2000) and Klump et al. (2007, 2008) have emphasized that the economic interpretation of the CES production technology requires care. In particular, they recommend "normalizing" CES technologies when analyzing the theoretical consequences of variation in the elasticity of substitution. The central argument is that variation in the elasticity of substitution can only be isolated by normalization. Since Klump and La Grandville (2000), in particular, it has often been argued that normalization is an essential device for examining the effects of variation in the elasticity of substitution. The normalized CES production function has since been used in theoretical work by Antony (2010), Cantore and Levine (2011), Growiec (2011), Irmen (2011), Miyagiwa and Papageorgiou (2007), Nakumura (2009), Papageorgiou and Saam (2008) and Wong and Yip (2010), among others. It has been adopted as a framework for empirical analysis in Antony (2008), Cantore et al. (2010), Klump et al. (2007, 2008), León-Ledesma et al. (2010a,b) and Mallick (2008). Much of this work is surveyed in Klump et al. (2011), who provide further references and make clear that the relevant literature is now extensive. Several of the papers cited above develop or reinterpret the idea of normalization, notably Cantore and Levine (2011).

This paper will argue that the benefits of explicit normalization have been exaggerated, especially for theoretical work. It will seek to distinguish between instances where it is useful to normalize a CES production function in the way that Klump and La Grandville (2000) and Klump et al. (2011) recommend, and instances where the idea could be misused. These dangers arise not from the logic of the procedure, but from its starting point—the idea, or assumption, that the effects of variation in the elasticity of substitution can be isolated in a meaningful way. As we will see, attempts to achieve this in practice will encounter some deep conceptual problems.

2. Normalization

It may seem odd that there is any normalization issue to raise at all. Perhaps the easiest way to demonstrate the underlying problem is to imagine a productivity comparison between two firms, with production functions $AF(K,L)$ and $BG(K,L)$ respectively. At first glance, the parameters A and B enter the production functions symmetrically and have the same interpretation, as TFP parameters. But since the production technologies differ, a direct comparison of the relative magnitudes of A and B has limited economic meaning. The two are not on the same scale, and the mathematical symmetry is misleading about the economic content of the comparison.

This is a simple illustration of a more general problem, which emerges especially clearly in the CES case. If the elasticity of substitution is allowed to vary, this is rather like moving from one function $F(K,L)$ to another, $G(K,L)$. This raises the issue of whether other technology parameters will retain the same economic interpretation as before, and what it means, in economic terms, to vary the elasticity of substitution while holding other parameters "constant". Different proposals for normalizing the CES technology are different proposals about what, exactly, should be held constant as the elasticity of substitution is varied. Cantore and Levine (2011) provide an especially useful discussion of the problem, emphasizing that technology parameters should be 'dimensionless constants', which are independent of the choice of units. But this, by itself, is not enough to ensure that variation in one parameter leaves the interpretation of others unchanged, as we will see below.

For simplicity, the discussion throughout will assume that there are just two inputs, capital and labor, and constant returns to scale. Most researchers who adopt CES use the standard (ACMS) form, due to Arrow et al. (1961):

$$Y = A(bK^{\rho} + (1-b)L^{\rho})^{\frac{1}{\rho}}$$

where Y, K and L are output, capital and labor respectively, and where the elasticity of substitution $\sigma = 1/(1-\rho)$. Much of the discussion that follows centers on assumptions about the TFP parameter A, and the distribution parameter b, for which the admissible range is $0 < b < 1$. La Grandville (1989) and some later authors argue that, when varying the elasticity of substitution, both A and b should be considered functions of the elasticity of substitution. They derive explicit relationships that can be used to normalize the function as the elasticity of substitution varies.

The easiest interpretation of normalization is to view the inputs of capital and labor as index numbers, so that each could be measured relative to arbitrarily-chosen benchmark values. We can then write the CES production function in the "calibrated share form" of Rutherford (1995):

$$Y = Y_0\left(\pi_0\left(\frac{K}{K_0}\right)^{\rho} + (1-\pi_0)\left(\frac{L}{L_0}\right)^{\rho}\right)^{\frac{1}{\rho}} \tag{1}$$

where the parameter π_0 corresponds to the capital share that arises at a benchmark capital-labor ratio K_0/L_0 and output per worker level Y_0/L_0, under perfect competition and marginal productivity factor pricing. The ACMS form can then be seen as

[2] Relevant work by theorists includes Lucas (1990), Laitner (1995) and Turnovsky (2002). The potential importance of CES for the analysis of short-run macroeconomics is emphasized by Cantore et al. (2010).