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Refining linear rational expectations models and equilibria

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ABSTRACT

This paper develops the case for forward convergence as a model refinement scheme for linear rational expectations models and an associated no-bubble condition as a solution selection criterion. We relate these two concepts to determinacy and characterize the complete set of economically relevant rational expectations solutions to the linear rational expectations models under determinacy and indeterminacy. Our results show (1) why a determinate solution is economically cogent in most, but not all, cases, and (2) that those models that are not forward-convergent have no economically relevant solutions.

1. Introduction

Recent macroeconomics literature has utilized the concept of determinacy as a primary criterion for characterizing the economic properties of a given rational expectations model and its solutions. While some researchers argue that determinacy is necessary and sufficient for a model to be economically relevant, others argue that multiple rational expectations solutions can be admissible as well in some cases of indeterminacy. Thus models and their solutions are not dismissed as implausible simply because they are indeterminate. Furthermore, determinacy is a criterion purely based on the number of stable solutions. Therefore, determinacy alone does not automatically warrant an economic plausibility of a given model and its determinate solution, as has been argued by Bullard and Mitra (2002), Cho and McCallum (2009), Honkapohja and Mitra (2004), and others.[2] Several solution selection criteria have been proposed to narrow down the set of relevant equilibria in indeterminate models

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[2] Here the word "determinacy" is being used in the sense that is standard in monetary economics and frequently utilized elsewhere, namely, to designate a model specification in which there exists only one rational expectations solution that is dynamically stable - literally a "single stable solution". (McCallum, 2012) has argued that this terminology is highly inappropriate, however, since the traditional meaning of the word "determinate" is that the model at hand clearly points to a single relevant solution. Thus the usage in question proceeds as if the single stable solution requirement was equivalent to the desired condition - i.e., that the model at hand provides a unique prediction as to the behavior of the (model) economy. A unique prediction is what "determinacy" is supposed to mean, however, so it is unsatisfactory for this word to be used as a synonym for the single stable solution condition, especially since there are examples in which it is clearly inappropriate. In the present paper we retain that usage, nevertheless, in order to facilitate communication.

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including, for example, the minimum state variable (MSV) criterion of McCallum (1983), McCallum (2007), the expectational stability criterion of Evans and Honkapohja (2001), and a fairly recent proposal by Driskill (2006).

This extent of disagreement over the role of determinacy may have been a consequence of the absence of any step for refining "models" to begin with. Here we propose a model refinement scheme for a general class of linear models, together with a solution selection criterion, and characterize the complete set of economically plausible equilibria under determinacy and indeterminacy on the ground of our criterion. In particular, we adopt the forward convergence and no-bubble conditions proposed by Cho and Moreno (2011), and relate these refinement schemes to determinacy by means of characterization results reported by McCallum (2007).

In their study of hyperinflation and monetary reform, Flood and Garber (1980) introduce the notion of "process consistency" and suggest that it is an essential characteristic of any model variable that pretends to serve as money. Specifically, in the context of a Cagan-type monetary model, they solve the model forward and argue that any "process inconsistent" money supply that explodes eventually will be rejected by the public because it does not provide a finite solution for the price level. In this setting, process consistency simply amounts to the case that a rational expectations model can be solved forward and they argue that any reasonable model should possess this minimal economic characteristic. (Blanchard and Kahn, 1980) also consider this principle in a class of models with lagged endogenous variables. When they derive the fundamental solutions, they state at the beginning of their Section 2 as "*we also require that expectations of X_t (predetermined) and P_t (non-predetermined) do not explode,… This in particular rules out "bubbles" of the sort considered by Flood and Garber (1980).*" However, their requirement is an assumption and thus it is not examined because they did not solve the model forward explicitly. This method of solving rational expectations models forward had not been developed and applied to more general models with lagged variables until Cho and Moreno (2011) developed the forward method for such models. Their forward convergence property, which we propose as a model refinement, amounts to a generalization of process consistency and here we argue that, indeed, any model that fails to satisfy the forward convergence condition has no cogent rational expectations solution.

The Cho and Moreno methodology also provides a solution selection criterion known as no-bubble condition in the class of the models that satisfy the forward convergence, and yields a well-known forward (forward-looking) solution in the sense of Blanchard (1979). When a model is solved forward, there remains a term involving the expectation of future endogenous variables, often called "a bubble term". A non-zero expectational term implies that agents' decision on current endogenous variables is influenced by this expectational effect in the future. It is well-known that the expectational term is not zero when evaluated with any non-fundamental solution, which depends not only on the state variables of the underlying model but also other variables outside the model. For this reason, this solution is often referred to as a bubble or sunspot solution. In contrast, a common understanding was that the expectational term should be zero when it is evaluated with a fundamental solution that depends only on the state variables of the model. Therefore, the absence of the expectational effect is a natural requirement for an equilibrium path to be characterized by the history of the state variables. Indeed, the literature has adopt this notion to select an equilibrium, and the resulting solution has been named as the forward solution. For example, using a simple asset bubble model, (Evans and Honkapohja, 2001) (pp. 220–221) actually define a fundamental solution by assuming that the bubble term is zero and show that the forward solution is the true equilibrium in their model. The transversality condition in infinite horizon models is also equivalent to the no-bubble condition in that the present discount value of capital stock should be zero.

The contribution of Cho and Moreno (2011) is that general models with lagged endogenous variables can also be solved forward and this assumption can be verified by the forward method. Their key result is that the no-bubble condition – zero expectational term – holds only for one fundamental solution and that all other fundamental solutions fail to satisfy it, even though they are technically referred to as "bubble-free". The solution that satisfies the no-bubble condition is named as the forward solution following the literature. This condition has been recently applied to find an equilibrium for a New-Keynesian model with bond pricing in Campbell et al. (2014). Accordingly, we contend that the no-bubble condition constitutes a relevant solution refinement scheme.

Cho and Moreno (2011), however, do not relate their refinement schemes to determinacy. Accordingly, it is our task here to derive this relationship, drawing on results of McCallum (2007), and to characterize all the rational expectations equilibria in relation to determinate and indeterminate models. In the process, we will also extend one of McCallum's results so as to apply to non-fundamental, as well as fundamental, solutions. The relationship between determinacy and our refinement schemes derived in this paper has three important implications. First, a determinate solution satisfies the no-bubble condition, thus coincides with the forward solution in the vast majority of macroeconomic models. As mentioned above, determinacy has been treated as a property for a well-defined economic model to possess in the literature although it is purely a statistical criterion. Therefore, our result provides an economic explanation for determinacy. Second, there does exist some cases, though rare, in which a determinate solution does not coincide with the forward solution. We show, through an example, that the determinate solution is hardly acceptable as an equilibrium in Section 5. Third, in the case of indeterminacy, we identify the conditions under which the forward solution does not exist. This class of models is ruled out on the ground of our model refinement, the forward convergence condition in the spirit of Flood and Garber (1980).

The paper is organized as follows. Section 2 presents a general class of linear rational expectations models and characterizes the set of rational expectations equilibria. In Section 3, necessary and sufficient conditions for determinacy are stated. In section 4, we formally define the concept of forward convergence and study the relation between determinacy and forward convergence. Section 5 classifies rational expectations models with these two properties and characterizes the full set of equilibria. In Section 6, we apply our methodology to an example based on a standard New-Keynesian model. Section 7 concludes.