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# Ant colony optimization theory: A survey

Marco Dorigo[a,*,1], Christian Blum[b,2]

[a] IRIDIA, Université Libre de Bruxelles, CP 194/6, Ave. F. Roosevelt 50, 1050 Brussels, Belgium
[b] ALBCOM, LSI, Universitat Politècnica de Catalunya, Jordi Girona 1-3, Campus Nord, 08034 Barcelona, Spain

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**Abstract**

Research on a new metaheuristic for optimization is often initially focused on proof-of-concept applications. It is only after experimental work has shown the practical interest of the method that researchers try to deepen their understanding of the method's functioning not only through more and more sophisticated experiments but also by means of an effort to build a theory. Tackling questions such as "how and why the method works'' is important, because finding an answer may help in improving its applicability. Ant colony optimization, which was introduced in the early 1990s as a novel technique for solving hard combinatorial optimization problems, finds itself currently at this point of its life cycle. With this article we provide a survey on theoretical results on ant colony optimization. First, we review some convergence results. Then we discuss relations between ant colony optimization algorithms and other approximate methods for optimization. Finally, we focus on some research efforts directed at gaining a deeper understanding of the behavior of ant colony optimization algorithms. Throughout the paper we identify some open questions with a certain interest of being solved in the near future.


*Keywords:* Ant colony optimization; Metaheuristics; Combinatorial optimization; Convergence; Stochastic gradient descent; Model-based search; Approximate algorithms

* Corresponding author. Tel.: +32 2 6503169; fax: +32 2 6502715.
*E-mail addresses:* mdorigo@ulb.ac.be (M. Dorigo), cblum@lsi.upc.edu (C. Blum).

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## 1. Introduction

In the early 1990s, ant colony optimization (ACO) [20,22,23] was introduced by M. Dorigo and colleagues as a novel nature-inspired metaheuristic for the solution of hard combinatorial optimization (CO) problems. ACO belongs to the class of metaheuristics [8,32,40], which are approximate algorithms used to obtain good enough solutions to hard CO problems in a reasonable amount of computation time. Other examples of metaheuristics are tabu search [30,31,33], simulated annealing [44,13], and evolutionary computation [39,58,26]. The inspiring source of ACO is the foraging behavior of real ants. When searching for food, ants initially explore the area surrounding their nest in a random manner. As soon as an ant finds a food source, it evaluates the quantity and the quality of the food and carries some of it back to the nest. During the return trip, the ant deposits a chemical pheromone trail on the ground. The quantity of pheromone deposited, which may depend on the quantity and quality of the food, will guide other ants to the food source. As it has been shown in [18], indirect communication between the ants via pheromone trails enables them to find shortest paths between their nest and food sources. This characteristic of real ant colonies is exploited in artificial ant colonies in order to solve CO problems.

According to Papadimitriou and Steiglitz [56], a CO problem $\mathcal{P} = (\mathcal{S}, f)$ is an optimization problem in which, given a finite set of solutions $\mathcal{S}$ (also called *search space*) and an objective function $f : \mathcal{S} \mapsto \mathbb{R}^+$ that assigns a positive cost value to each of the solutions, the goal is either to find a solution of minimum cost value, [3] or—as in the case of approximate solution techniques—a good enough solution in a reasonable amount of time. ACO algorithms belong to the class of metaheuristics and therefore follow the latter goal. The central component of an ACO algorithm is a parametrized probabilistic model, which is called the *pheromone model*. The pheromone model consists of a vector of model parameters $\mathcal{T}$ called *pheromone trail parameters*. The pheromone trail parameters $\mathcal{T}_i \in \mathcal{T}$, which are usually associated to components of solutions, have values $\tau_i$, called *pheromone values*. The pheromone model is used to probabilistically generate solutions to the problem under consideration by assembling them from a finite set of solution components. At runtime, ACO algorithms update the pheromone values using previously generated solutions. The update aims to concentrate the search in regions of the search space containing high quality solutions. In particular, the reinforcement of solution components depending on the solution quality is an important ingredient of ACO algorithms. It implicitly assumes that good solutions consist of good solution components. [4] To learn which components contribute to good solutions can help assembling them into better solutions. In general, the ACO approach attempts to solve an optimization problem by repeating the following two steps:

- candidate solutions are constructed using a pheromone model, that is, a parametrized probability distribution over the solution space;
- the candidate solutions are used to modify the pheromone values in a way that is deemed to bias future sampling toward high quality solutions.

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[3] Note that minimizing over an objective function $f$ is the same as maximizing over $-f$. Therefore, every CO problem can be described as a minimization problem.

[4] Note that this does not require the objective function to be (partially) separable. It only requires the existence of a fitness-distance correlation [41].