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The plurality problem with three colors and more☆,☆☆

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Abstract

The *plurality problem* is a game between two participants: Paul and Carole. We are given n balls, each of them is colored with one out of c colors. At any step of the game, Paul chooses two balls and asks whether they are of the same color, whereupon Carole answers yes or no. The game ends when Paul either produces a ball a of the plurality color (meaning that the number of balls colored like a exceeds those of the other colors), or when Paul states that there is no plurality. How many questions $L_c(n)$ does Paul have to ask in the worst case?

For $c = 2$, the problem is equivalent to the well-known *majority problem* which has already been solved (Combinatorica 11 (1991) 383–387). In this paper we show that $3\lfloor n/2\rfloor - 2 \leqslant L_3(n) \leqslant \lfloor 5n/3\rfloor - 2$. Moreover, for any $c \leqslant n$, we show that surprisingly the naive algorithm for the plurality problem is asymptotically optimal.

Keywords: Combinatorial search; Algorithm analysis; Majority problem; Game

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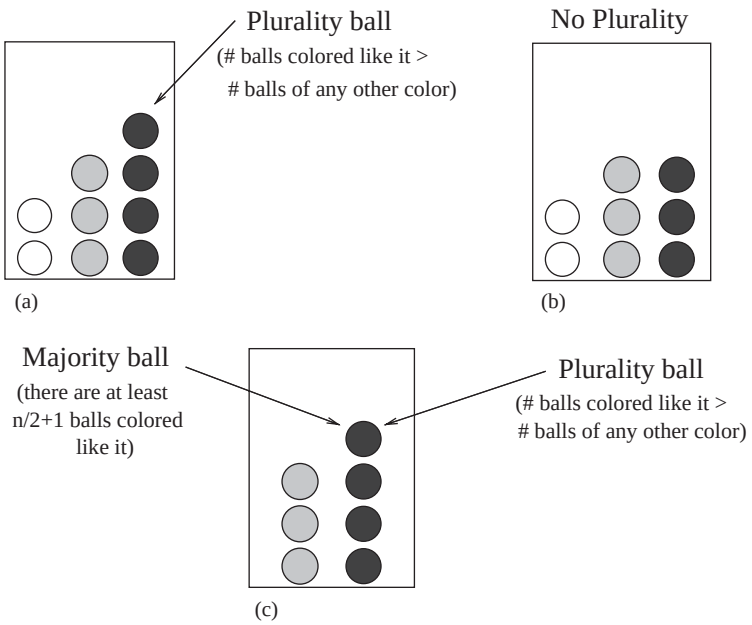

Fig. 1.

1. Introduction

The plurality problem can be stated as a game between two players: Paul and Carole. There are n balls, each of them colored with one out of c colors. The *plurality color* is the color that has been used the most, i.e., such that the balls colored with it strictly outnumber the balls of any other color. A *plurality ball* is any ball colored with the plurality color (see Fig. 1a). Note that a plurality color (and ball) not always exists (see Fig. 1b).

At any step of the game, Paul chooses two balls and asks whether they are of the same color, whereupon Carole answers yes or no. The game ends when Paul either produces a ball of the plurality color, or when Paul states that there is no plurality. How many questions $L_c(n)$ does Paul have to ask in the worst case?

This problem is a generalization of the well-known *majority problem*, a classical problem in Combinatorial Search [1], in which we are given n balls and two colors, e.g., white and black. The aim is to produce a ball of the *majority color* (meaning that the number of balls with that color is strictly greater than that of the other color), or to state that there is no majority (this happens when there is the same number of white and black balls). The *majority problem* asks to determine how many questions Paul needs in the worst case. It is straightforward to observe that the plurality problem with two colors is equivalent to the majority problem (see Fig. 1c).

This kind of problems finds several interesting applications in the field of fault diagnosis of multiprocessor systems introduced in [10].