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Capital–labor substitution and balanced growth

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ABSTRACT

This paper explores how different values of the elasticity of substitution between capital and labor (σ) impact on the balanced growth paths and growth potential of a country in exogenous growth models. The behavior of the system depends on the value of σ and the passage of σ through two critical values causes a qualitative change in the nature of the singular points and of its trajectories. The balanced growth path defined by a singular point in the form of a saddle-path exists and is locally stable if σ lies between two critical values.

1. Introduction

Exogenous growth models[1] are usually characterized by the notion that no long-run growth is possible without technical progress. Due to diminishing factor returns combined with constant returns to scale, the capital–labor ratio and per capita output settle down to some steady state level. However, this result is due mainly to the inappropriate use of the Cobb–Douglas production function in growth models in which the elasticity of substitution between capital and labor (σ) is restricted to be unity.[2] Although the channel of σ for fending off the diminishing factor returns received some sporadic attention over the past 50 years, de La Grandville (1989) was the first to systematically explore the relationship between σ and economic growth.[3] Subsequent investigations by Klump and de La Grandville (2000) and Klump and Preissler (2000), among others, have significantly contributed to understand the importance of σ in economic growth. One major contribution made by these authors is to institute normalization of the CES production function to investigate the role of σ in growth models in a meaningful way. Recently, de La Grandville and Solow (2009) have shown that in the Solow growth model, an economy can grow indefinitely even without technical progress if the value of σ is large enough to exceed some critical value.

This paper contributes to this line of research in several ways by studying how σ impacts on the steady state and the balanced growth path in exogenous growth models. First, as a simple extension to de La Grandville and Solow (2009), it explores the possibility of perpetual slowdown in the Solow growth model conditional on another critical value of σ. This possibility

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[1] By exogenous growth model, we mean a model in which technical progress is exogenously determined. Both the models of Solow (1956), Cass (1965), Koopmans (1965) fall into this category.

[2] Many argue that the constancy of observed factor shares is the main reason for using the Cobb–Douglas production function in growth models. A value of σ equal to unity is sufficient but not a necessary condition for this constancy. A value of σ less than unity combined with labor-augmenting technical change can also ensure constant factor shares.

[3] Solow (1956, pp. 77–78) and Pitchford (1960) were the first to raise the possibility of perpetual growth.

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of perpetual slowdown is not new in the literature; we provide a critical value of σ for such slowdown. Second, it determines the conditions that need to be satisfied for each critical value of σ to have an economic interpretation. Finally, it shows that the above results also carry over to the Ramsey model with consumer optimization, and analytically examines how the existence of the steady state and the balanced growth path depend on the value of σ.

The pivotal role of σ in economic growth can be perceived intuitively. The higher is σ, the greater the similarity between capital and labor in the production function. Therefore, the incremental capital is easily substituted for labor that leads to a nearly equiproportionate increase in both factors, which, under constant returns to scale, resists the pull of diminishing returns to capital (Brown, 1968, p. 50). If $\sigma > 1$, one of the Inada conditions – marginal and average products of capital tend to zero when capital–labor ratio approaches infinity – is violated. de La Grandville and Solow (2009) demonstrate that if σ exceeds a critical value that is greater than 1 (σ_H^c), perpetual growth is possible even without technical progress. This critical value depends on structural parameters such as saving, population growth, depreciation rates, and capital share. We argue that a country may not satisfy conditions for its critical value of σ to be greater than unity. Rather its critical value may lie between 0 and 1 (σ_L^c). An economy can experience perpetual slowdown if its σ falls below σ_L^c. The expressions for σ_L^c and σ_H^c are the same but they assume values greater or less than 1 conditional on values of the structural parameters. Since countries differ in these parameters, each country will have at most one critical value (σ^c). We interpret σ^c as the growth potential of a country, and actual σ, which characterizes production, as the capability to realize that potential. We also encounter a third possibility in which σ^c becomes negative. Since σ is non-negative by definition, such a critical value implies that a country does not possess the potential for perpetual growth or risk of perpetual slowdown.

We then investigate the possibility of perpetual growth and slowdown in the Ramsey model. We analytically examine the steady state and balanced growth path for different values of σ. If $\sigma_L^c < \sigma < \sigma_H^c$, the linearized system of two differential equations (one for the growth rate of capital–labor ratio and another for per capita consumption) has one positive and one negative eigenvalue, and thus the balanced growth path defined by a singular point in the form of a saddle-path exists and is locally stable. Per capita output, capital stock, and consumption settle down to some steady state level. On the other hand, if $\sigma > \sigma_H^c$ or $\sigma < \sigma_L^c$, the determinant of the characteristic matrix of the linearized system becomes zero. The integral curves are straight lines, which no longer possess a singularity (Gandolfo, 1997) and no steady state equilibrium exists. Thus, the behavior of the system depends on the value of σ and the passage of σ through critical values causes a qualitative change in the nature of the singular point and of its trajectories. These critical values of σ are the points of Hopf bifurcation (Gandolfo, 1997). However, the system does not give rise to limit cycles.

To get a sense of the growth potential of the countries, we calibrate σ^c using country level data and plausible choice of parameter values. We find that the majority of countries in our sample have a negative σ^c. A few countries have a bright growth potential as they have σ_H^c, and several countries predominantly from Africa have σ_L^c showing their vulnerability. However, these results are very sensitive to the choice of the depreciate rate.

The rest of the paper proceeds as follows. A brief literature review on the implications of σ for growth theory is provided in Section 2. Section 3 discusses Solow model with CES production function and shows possibilities of perpetual growth and slowdown. Ramsey model with CES production function is introduced in Section 4. The dependence of the steady state and the balanced growth path on σ is also discussed in this section. Section 5 calibrates σ^c at the country level. Finally, Section 6 concludes.

2. Relevant literature

Many important growth issues depend on the precise value of σ such as the possibility of perpetual growth or decline, the growth rate and level of steady state income per capita, the speed of convergence to the steady state, and the relative role of productive factors and technical efficiency in explaining differences in per capita income.

Klump and de La Grandville (2000) show that if two countries start with identical initial conditions (capital–labor ratio, saving and population growth rates), then at any stage of development including the steady state, if it exists, the country with a larger value of σ will experience higher per capita income. de La Grandville (1989) has shown that, at any stage of an economy's development, the growth rate of per capita output is increasing with the value of σ.[4] He argues that a larger value of σ entails higher transformation rates between sectors of different factor intensity. When one activity is decreasing to the benefit of the other, the increase in production in the second sector can be made larger if the value of σ is high. Yuhn (1991) conjectures that the higher value of σ for South Korea may be a reason for its faster growth compared to the United States, which has a low σ. Mallick (2008) tests the de La Grandville (1989) hypothesis at the cross-country level and finds strong support for it. Turnovsky (2002) demonstrates quantitatively how convergence of per capita output to its steady state level is negatively related to the value of σ.

Papageorgiou and Saam (2008) show that in the Solow model with three-factor nested CES production function, the sufficient condition for a steady state is achieved for a wide range of substitution parameters, which is in contrast with the two-factor Solow model where only σ equals to one is by itself sufficient to guarantee the existence of a steady state. In the Diamond model, on the other hand, multiple equilibria can occur when σ is lower than the capital share. Miyagiwa and Papageorgiou (2003) show that in the Diamond model if σ is sufficiently high, a further increase in σ lowers output

[4] This result also holds in the presence of factor-augmenting technical change (de La Grandville and Solow, 2009).