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The Ramsey model with AK technology and a bounded population growth rate

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ABSTRACT

This paper generalizes the Ramsey AK model by allowing the population growth rate to be variable over time subject only to be between prescribed upper and lower limits. Contrary to the standard AK setting, convergence can occur. Moreover, monotonicity as well as an asymptotic balanced growth path equilibrium may arise in the model.

1. Introduction

The Ramsey (1928) growth model is a basic model in macroeconomics that develops the standard Solow (1956) growth model by taking into account an endogenous determination of the level of savings. His ideas were later taken up independently by Cass (1965) and Koopmans (1965), and have now become a major workhorse model in modern macroeconomics. It is known that usually standard economic growth theory assumes that population grows exponentially (Malthusian model, 1798). However, this population model describes infinite growth without constraints, which is not a realistic hypothesis. In practice, growth is limited by available resources. By including an additional quadratic term with a negative coefficient in the exponential model, Verhulst (1838), working also on population growth, proposed a model containing an auto-limitation term that represents some theoretical limiting resource. In this alternative model, known as logistic model, the population stock evolves according to an elongated S-curve, which has a point of inflection at the maximal value of the growth rate, and then levels off at a new but higher plateau, at which point the growth rate declines to zero. Recent forecasts (United Nations, 2000), having the world as unit of analysis, confirm that the annual growth rate of population is expected to fall gradually until 2100, and that world population will stabilize at a level of about 11 billion people by 2200 . Thus, not only theoretically but also empirically, it seems reasonable to model population size as following a logistic process. The role of variable population growth rate in dynamic models has recently attracted a great attention in the literature. Accinelli and Brida (2007) have analyzed the dynamics of the Ramsey model with neoclassical technology and a logistic-type evolution over time for population size. Guerrini (2006, forthcoming-a, forthcoming-b) and Ferrara and Guerrini (2008) have extensively explored the standard Solow–Swan, Ramsey and Uzawa growth models under the assumption of logistic growth or bounded population growth rate. Finally, Bucci and Guerrini (2009) have assumed, within the Solow–Swan growth model, a linear AK aggregate production function without diminishing returns to capital and a logistic-type population law. The novelty of their work lies in the possibility to produce transitional dynamics in the standard AK framework with exogenous savings rate. As it is

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well-known (e.g., Barro and Sala-i-Martin, 2004), the standard AK endogenous growth model displays no transitional dynamics. The main objective of our paper is to generalize their result investigating whether and, eventually, how a bounded population growth rate hypothesis might affect the dynamics of a standard Ramsey model with linear aggregate technology. Notice that the underlying optimization problem that produces the bounded population growth model is an interesting question in its own right. In this kind of setup, contrary to the standard Ramsey model, we prove that per capita consumption and per capita physical capital are not necessarily always growing. The model exhibits transitional dynamics with convergence that may be monotonic. As well, an asymptotic balanced growth path equilibrium with consumption and capital growing asymptotically with constant rates of growth may arise. A final comment. There have been different ways to produce transitional dynamics within the AK world. Acemoglu and Ventura (2002), Boucekkine et al. (2005), Carroll et al. (1997), Gomez (2008), Kocherlacota and Yi (1995, 1996), and Tamura (1991, 1996, 2002, 2006) are among the most comprehensive and rigorous examples. For example, Tamura (2002) develops a model of economic and population growth where human capital accumulation causes the economy to switch from agriculture to industry endogenously. For low human capital, the classical method dominates the industrial one, but for high human capital the opposite is true. The switch to the industrial mode of production is made possible by human capital accumulation. However, perpetual human capital investment eventually raises the productivity of the industrial technology above the classical one. Upon the switch, population growth and human capital accumulation accelerates. In conclusion, there are many alternative ways to produce transition dynamics in something like an AK model. However, none of the above mentioned works deals with the idea of variable population, which, thus, represents the novelty of this paper.

2. The Ramsey model with AK technology and constant population growth

Consider a closed economy where many structurally identical rational agents provide labor services purchase goods for consumption and save. Each individual in this economy works and supplies inelastically one unit of labor services per unit of time. These two assumptions together imply that population size equals the aggregate labor-force, i.e., the total number of workers. Per capita and per worker variables are therefore the same. Population L_t rises over time at the constant rate n, i.e., $L_t = e^{nt}, L_0 = 1$. Consumption goods, or aggregate output, Y_t, are produced with a linear aggregate technology employing solely capital K_t as an input, i.e., $Y_t = AK_t$, where $A > 0$ is constant. As it is well-known, this production function differs from the neoclassical one in that the marginal product of capital is not diminishing, $Y''_t = 0$, and the Inada conditions are violated. In particular, $\lim_{K_t \to 0} Y'_{K_t} = \lim_{K_t \to \infty} Y'_{K_t} = A > 0$. As in Rebelo (1991), the assumption of global absence of diminishing returns to capital can be motivated by thinking of K_t as broadly encompassing not only physical, but also human, knowledge, infrastructure capital, and so on. Since agents are identical to each other we can focus on the choices of a single representative infinitely-lived individual whose discounted intertemporal utility U, derived from the consumption of a single good, is given by

$$\int_0^\infty u(c_t) e^{-\rho t}\, dt, \tag{1}$$

where $c_t \equiv C_t / L_t$ is the individual stream of consumption, or per capita consumption, $u(c_t)$ is the instantaneous utility function of the representative agent, and $\rho > 0$ is the pure rate of time preference, or subjective discount rate. As usual, we assume that $u(c_t)$ is increasing and concave in c_t and satisfies the Inada conditions: $\lim_{c_t \to 0} u'_{c_t} = \infty, \lim_{c_t \to \infty} u'_{c_t} = 0$. Let us assume that $u(c_t)$ is of the CIES-form (constant intertemporal elasticity of substitution), so that Eq. (1) can be recast as

$$\int_0^\infty \left(\frac{c_t^{1-1/\sigma} - 1}{1 - 1/\sigma} \right) e^{-\rho t}\, dt, \tag{1'}$$

with $\sigma > 0$ denoting the constant intertemporal elasticity of substitution in consumption. The logarithmic utility $u(c_t) = \ln c_t$ is a particular case of the CIES instantaneous utility function when $\sigma = 1$. The income which is not consumed is used to accumulate additional capital. Hence, at the aggregate level we have

$$\dot{K}_t = Y_t - C_t, \quad K_0 > 0,$$

where $\dot{K}_t$ denotes (gross and net) aggregate capital investment and C_t is total flow of consumption. Notice that the depreciation rate of capital is set equal to zero, a simplification that affects none of our major results. Given the expression above, the law of motion of per capita capital, $k_t \equiv K_t / L_t$, reads as

$$\dot{k}_t = (A - n)k_t - c_t. \tag{2}$$

The representative agent intertemporal optimization problem consists in maximizing (1′), subject to the individual budget constraint (2), the non-negativity constraints on c_t and k_t, and the initial per capita capital holding, $k_0 > 0$. Application of the Pontryagin's maximum principle to this optimal control problem leads to

$$\dot{k}_t = (A - n)k_t - c_t, \tag{3}$$
$$\dot{c}_t = \sigma(A - n - \rho)c_t, \tag{4}$$