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Equilibrium stability in open economy models ☆

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ABSTRACT

This paper derives analytical results for the relationship between the slope of the excess demand function and the dynamic properties around a deterministic steady state in a two country model. In models that admit multiple steady states, the sign of the slope of the excess demand function is positive for some steady states and negative for others. To obtain stationarity of the net foreign asset position under incomplete financial markets I introduce the stationary inducing devices analyzed in Schmitt-Grohé and Uribe (2003) and Ghironi (2006). For portfolio costs, a debt-elastic interest rate, or an overlapping generations framework the equilibrium dynamics around a steady state are unbounded if the excess demand function for the foreign traded good is increasing in the good's own price. Otherwise the dynamics are bounded and locally unique. By contrast, with Uzawa-type preferences, the equilibrium dynamics around a steady state are shown to be bounded and locally unique irrespective of the sign of the slope of the excess demand function.

1. Introduction

As shown in Bodenstein (2011), models of the international business cycle admit multiple locally isolated deterministic steady states, if the elasticity of substitution between traded goods is sufficiently low.[1] For the case of a model with two agents and two goods, standard theorems in general equilibrium theory imply that the slope of a good's excess demand as a function of its relative price needs to change sign for multiple steady states to exist.

This paper derives analytical results for the relationship between the slope of the excess demand function and the dynamic properties around a deterministic steady state in a two country model. Each country produces a distinct traded good that serves as a limited substitute for the other country's good. Labor is the only factor of production and international financial markets are incomplete with a non-state-contingent bond being the only asset that trades internationally.

In the standard model with incomplete markets the steady state is undetermined since the growth rate of marginal utility does not depend on the allocation of bond holdings. Absent arbitrage opportunities, the price of the non-state-contingent bond is equalized across countries implying that expected marginal utility growth is equalized across countries. In the

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[1] Consider an endowment economy with two countries and two traded goods that are imperfect substitutes as in Mas-Colell et al. (1995). The countries are mirror images of each other with respect to preferences and endowments. One equilibrium always features a relative price of the traded goods equal to unity. However, there can be two more equilibria if there is home bias in consumption and the price elasticity of substitution between goods is low. If the price of the domestic good is high relative to the price of the foreign good, domestic agents are wealthy compared to the foreign agents. If the elasticity of substitution is low, foreigners are willing to give up most of their good in order to consume at least some of the domestic good, and domestic agents end up consuming most of the domestic and the foreign good. The reverse is true as well. Foreign agents consume most of the goods, if the foreign good is expensive in relative terms. Of course, these last two scenarios cannot be an equilibrium for high values of the elasticity of substitution. In the limiting case of perfect substitutability.

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deterministic steady state, this condition contains no information about the steady state values of the system and the system of equilibrium conditions becomes underdetermined. Any level of international bond holdings is a steady state. Furthermore, net foreign asset position follows a unit root process when the model is linearized around a deterministic steady state.

To obtain stationarity of the net foreign asset position in this setting, I introduce the stationary inducing devices analyzed in Schmitt-Grohé and Uribe (2003) and Ghironi (2006): a portfolio costs, a debt elastic interest rate, endogenous discounting, and an overlapping generations structure. All four approaches analyzed here, resolve this indeterminacy by construction. Stationarity of international bond holdings follows as agents finance additional consumption out of a positive net foreign asset position and the economy moves towards its steady state.

This paper studies the local dynamic properties under these stationarity inducing devices for different signs of the slope of the excess demand function of the foreign traded good. If agents face a portfolio cost for holding/issuing bonds, the equilibrium dynamics around a given steady state are unbounded (or unstable) if the excess demand function for the foreign traded good is increasing in the good's own price in this steady state. However, if the excess demand function for the foreign traded good is decreasing in the foreign goods price, the equilibrium dynamics are bounded and locally unique (saddle-path stable). The same results apply for models with a debt-elastic interest rate or an overlapping generations framework of Ghironi (2006) and Weil (1989). By contrast, if preferences allow for an endogenous discount factor as in Uzawa (1968), the equilibrium dynamics around a steady state are shown to be bounded and locally unique irrespective of the sign of the slope of the excess demand function.

Bodenstein (2011) also studies the relationship between equilibrium dynamics in the neighborhood of a steady state and the sign of the slope of the excess demand function. Whereas Bodenstein (2011) considers a richer class of models, all the results in that paper are only numerical. Here I derive with paper and pencil the exact nature of the relationship under investigation.

The remainder of the paper is organized as follows. Section 2 presents the static model that underlies the analysis, proves the existence of the equilibrium, and relates the sign of the slope of the excess demand function to steady state multiplicity. In Section 3, the static model is extended to incorporate dynamics and I analyze the characteristics of the steady states and the local dynamics under the different stationarity inducing approaches. Section 4 offers concluding remarks.

2. Model with multiple equilibria

Each country produces one good that can be traded internationally without frictions. The two goods are assumed to be imperfect substitutes in the household's utility function. Labor, which is supplied endogenously, is the sole factor of production and the population size is normalized to one. At this first stage, the model is static.

Households maximize utility subject to the budget constraint

$$\max_{\substack{c_{i1}, c_{i2} \\ c_i, l_i}} U(c_i, l_i), \tag{1}$$

$$\text{s.t. } \overline{P}_1 c_{i1} + \overline{P}_2 c_{i2} \leqslant \overline{P}_i w_i l_i + \overline{P}_i \Pi_i + dW_i, \tag{2}$$

where c_i is given by a linear-homogeneous aggregator $H_i(c_{i1},c_{i2})$.[2] H_i is assumed to satisfy

$$H_{ij} = \frac{\partial H_i}{\partial c_{ij}} > 0, \quad H_{iii} = \frac{\partial^2 H_i}{\partial c_{ii}^2} < 0, \quad H_{iji} = \frac{\partial^2 H_i}{\partial c_{ij} \partial c_{ii}} > 0,$$

and the Inada conditions

$$\lim_{c_{i1}\to 0} H_{i1}(c_{i1}, c_{i2}) = \lim_{c_{i2}\to 0} H_{i2}(c_{i1}, c_{i2}) = \infty,$$
$$\lim_{c_{i1}\to \infty} H_{i1}(c_{i1}, c_{i2}) = \lim_{c_{i2}\to \infty} H_{i2}(c_{i1}, c_{i2}) = 0.$$

The strictly concave period utility function $U(c,l)$ satisfies

$$U_c > 0, U_l < 0 \text{ and } U_{cc} < 0, \quad U_{ll} < 0, \quad U_{cl} \leqslant 0,. \tag{3}$$

c_i denotes final consumption, l_i labor, c_{ij} is the consumption of good j by a household located in country i. $\overline{P}_i$ is the price at which good i is traded and w_i is the wage in country i denoted in units of country i's traded good. Real profits are Π_i. dW_i is an exogenous lump sum transfer to agents in country $i$with $dW_1 + dW_2 = 0$. This lump sum transfer is introduced to make the derivations easily applicable to the dynamic economy introduced in the Section 3.

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[2] An aggregator that satisfies the restrictions imposed on H_i is given by the generalization of the *CES* aggregator as suggested by Dotsey and King (2005):

$$\frac{\alpha_{i1}}{(1+\eta)\rho}\left[\frac{(1+\eta)}{\alpha_{i1}}\left(\frac{c_{i1}}{c_i}\right) - \eta\right]^{\rho} + \frac{\alpha_{i2}}{(1+\eta)\rho}\left[\frac{(1+\eta)}{\alpha_{i2}}\left(\frac{c_{i2}}{c_i}\right) - \eta\right]^{\rho} = \frac{1}{(1+\eta)\rho}.$$

This aggregator allows for the elasticity of substitution to be non-constant. For $\eta = 0$, one obtains the standard *CES* aggregator.