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Can learning cause shorter delays in reaching agreements?☆

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ABSTRACT

This paper uses a continuous-time war of attrition model to investigate how learning about private payoffs affects delays in reaching agreement. At each point in time, players may receive a private Poisson signal that completely reveals the concession payoff to be high (good-news learning) or low (bad-news learning). In the good-news model, the expected delay is always non-monotonic in the learning rate: an increase in the learning rate prolongs delay in agreement if the learning rate is sufficiently low. In the bad-news model, numerical examples suggest learning prolongs delay as well.

1. Introduction

This paper develops a continuous-time war of attrition model with learning to investigate how learning affects delays in reaching agreement. In the model, two risk-neutral players decide when to concede. A player receives a high deterministic payoff if her opponent concedes first, while there is uncertainty about her payoff if she concedes first. As long as no player concedes, each player has a chance of privately learning whether her concession payoff is high or low (normalized to 0). For technical tractability, learning is modeled in a very stylized way: at each point in time, each player may receive a private Poisson signal that completely reveals the concession payoff. This paper focuses on two different ways of interpreting the Poisson signal. In the good-news (resp. bad-news) model, the signal reveals a high (resp. low) concession payoff, which increases (resp. decreases) the receiver's incentive to concede. For further simplicity, we also assume full symmetry between the two players; hence, we can focus on symmetric equilibrium of the game as in Bishop and Cannings (1978).

In the presence of learning, the war of attrition starts as a complete information game, but due to learning, incomplete information about the payoffs may develop over time. Due to the Poisson signal structure, a player can be either informed or uninformed about her concession payoff at any point of time, and one type of player is more willing to concede than the other. This enables us to fully characterize the unique symmetric equilibrium of the game.

The paper first compares the uninformed benchmark, in which there is no revelation of the concession payoff, with the full-information benchmark, in which there is immediate revelation of the concession payoff. It is shown that the expected delay is always shorter in the full-information benchmark. Since these two benchmarks correspond to the special cases in which the learning rate is 0 and infinity, respectively, one may conjecture that an increase in the learning rate always leads to shorter delays in reaching agreements. This conjecture is shown to be incorrect: the expected delay can be non-monotonic in the learning rate, and learning can cause longer delays in reaching agreements.

In the good-news model, we show that, at any point in time, equilibrium play falls into one of three possible cases. In the first case, an uninformed player randomizes between conceding and staying, while an informed player strictly prefers conceding immediately. In the second case, an uninformed player strictly prefers staying, while an informed player randomizes between conceding and staying. In the third case, an uninformed player strictly prefers staying, while an informed player strictly prefers conceding immediately. The equilibrium characterization depends on the expected learning rate. When the expected learning rate is relatively small, the game always stays in the first case. When the expected learning rate is intermediate, the game is initially in the third case, and

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switches to the first case if no player has received the Poisson signal for a sufficiently long time. When the expected learning rate is very high, the game starts out in the second case and eventually switches to the first case. Compared to the uninformed benchmark, learning causes longer (resp. shorter) delays when the learning rate is sufficiently low (resp. high).

In the good-news model, we conduct comparative statics analysis with respect to the learning rate. When the learning rate is very low, the average expected concession rate is determined by the indifference condition of the uninformed players, who become less optimistic about the concession payoffs as the learning rate goes up. Although an increase in the learning rate results in more informed players, who will concede immediately, the expected concession rate has to be lower in equilibrium in order to make the less optimistic uninformed players indifferent between conceding and staying. As a result, the expected delay is increasing in the learning rate. Due to this negative effect, an increase in the learning rate has no impact on welfare (in terms of a player's expected equilibrium payoff at the start of the game) when the learning rate is sufficiently low.

In contrast, when the learning rate is very high, a higher learning rate decreases the expected delay, and thus increases the welfare. Here, an increase in the learning rate has two opposite effects on the average expected concession rate. First, it leads to a higher average expected concession rate by increasing the chance of getting informed, since the informed players have the highest expected concession rate. Second, it leads to a lower average expected concession rate by making the uninformed players more reluctant to concede. When the learning rate is sufficiently high, a higher learning rate leads to a higher average expected concession rate by making the distribution of posterior beliefs more dispersed.

In the bad-news model, there is always one unique equilibrium pattern because the uninformed are more willing to concede than the informed players who have received bad-news signals. Initially, an uninformed player randomizes between conceding and staying, while an informed player strictly prefers staying. If both players receive the Poisson signal before conceding, the game eventually switches to a war of attrition between the informed players.

Different from the good-news model, the uninformed players obtain an additional value from receiving the Poisson signal in the bad-news model. In the good-news model, the continuation value after receiving the Poisson signal is always the high concession payoff, since the informed players always concede first. However, in the bad-news model, this continuation value is strictly larger than zero (the low concession payoff), since the informed benefit from the concession of the uninformed players. Because of this positive learning value, the uninformed players are more reluctant to concede. We show by example that in the bad-news model, learning leads to a longer expected delay compared to the uninformed benchmark.[1] Moreover, an increase in the learning rate has no impact on welfare in the bad-news model, since the uninformed players are always indifferent at the beginning of the game.

In the literature, continuous-time wars of attrition have been studied under both complete information (Hendricks et al., 1988) and incomplete information (Abreu and Gul, 2000; Damiano et al., 2010, 2012; Ponsati and Sakovics, 1995).[2] This paper extends the literature by considering a situation in which incomplete information is endogenously caused by learning. The game starts as a complete information game, and incomplete information about the payoffs develops over time.

The remainder of this paper is organized as follows. Section 2 presents the concession game. Sections 3.1 and 3.2 analyze the uninformed benchmark and the full-information benchmark, respectively. Section 4 characterizes the symmetric equilibria of this war of attrition under the good-news model, and Section 5 discusses how the learning rate affects expected delay and welfare. Section 6 contains the equilibrium characterization and comparative statics results under the bad-news model. Section 7 concludes the paper.

2. The game

2.1. Model setting

Consider a continuous-time war of attrition with two risk-neutral players ($i = 1, 2$) without discounting. Both players decide when to concede. As long as neither player concedes, the game continues with each player incurring a flow cost c, which reflects the cost of delay. The game ends when one of the players concedes. The players' lump-sum payoffs in this event are specified in the following matrix:

1 \ 2	*Stay*	*Concede*
Stay	$-, -$	$v_H, \ v_2$
Concede	v_1, v_H	$M, \ M$

If player i stays while her opponent $-i$ concedes, then player i is the winner of the game and gets a winning payoff of v_H. If player i concedes first, then she is the loser and gets a concession payoff v_i. The payoff when both players concede simultaneously is M. It is common knowledge that $M < v_H$, but there is incomplete information about concession payoffs v_1 and v_2. In particular, we assume that v_1 and v_2 are independently and identically drawn from a binary distribution: v_i can be either a positive number $v_L < v_H$ or zero. Each player i initially does not know the exact value of v_i. It is common knowledge that $v_i = v_L$ with prior probability p_0.

Following Keller et al. (2005) and Weng (2015), we introduce learning by assuming that as long as no player concedes, each player receives an exogenous private signal which is distributed as the first arrival time of a Poisson process. The Poisson processes are independent across players, and for simplicity, we assume that the arrival of the Poisson signal completely resolves uncertainty about v_i. In the good-news model, the arrival rate is λ if $v_i = v_L$ and zero otherwise. After receiving this good-news signal, player i assigns probability one to the event that $v_i = v_L$. Absence of the signal will make the player increasingly pessimistic about the probability that $v_i = v_L$. In the opposite bad-news model, the arrival rate is λ if $v_i = 0$ and zero otherwise. After receiving this bad-news signal, player i assigns probability one to the event that $v_i = 0$. Absence of the signal will make the player increasingly optimistic about the probability that $v_i = v_L$.

2.2. Strategies and equilibrium

At any point in time, a player is either informed or uninformed about her concession payoff. We refer to the former as an informed player, and the latter as an uninformed player.

A strategy for the uninformed player i is a cumulative distribution function $X^i : \mathbb{R}_+ \to [0, 1]$, where $X^i(t)$ denotes the probability that player i concedes to her opponent $-i$ by time t (inclusive).[3]

[1] In an earlier version of the paper, we show the opposite result when the positive learning value is absent.

[2] The game is a special case of the more general quitting games in Solan and Vieille (2001) and timing games in Laraki et al. (2005).

[3] This definition of strategy avoids the well-known problem in continuous-time games in which well-defined strategies may not lead to well-defined outcomes, as shown by Bergin and MacLeod (1993) and Simon and Stinchcombe (1989).