Journal of Mathematical Economics 60 (2015) 81–88

Contents lists available at ScienceDirect

Journal of Mathematical Economics

journal homepage: www.elsevier.com/locate/jmateco

Stochastic models for risky choices: A comparison of different axiomatizations

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ARTICLE INFO

Article history:
Received 3 May 2014
Received in revised form
22 April 2015
Accepted 24 June 2015
Available online 2 July 2015

Keywords:
Stochastic choice under uncertainty
Random tastes
Bounded rationality
Independence from irrelevant alternatives

ABSTRACT

For a long time researchers have recognized the need for applying stochastic models for analyzing data generated from agents' choice under risk. This paper compares and discusses recent axiomatizations of stochastic models for risky choice given by Blavatskyy (2008) and Dagsvik (2008). We show that some of Blavatskyy's axioms are equivalent to some of Dagsvik's axioms. We also propose new axioms and derive their implications. Specifically, we show that some of the results of Dagsvik (2008) can be derived under weaker axioms than those he proposed originally.

1. Introduction

It has been recognized at least since the work of Thurstone for almost a century ago that individuals in identical repetitions of the same choice setting often make seemingly inconsistent choices, cf. Thurstone (1927a,b), Tversky (1969), Luce and Suppes (1965), Suppes et al. (1989), Fishburn (1998) and Hey (2001). Thus, a central motivation for stochastic theories of choice is that agents may make errors when they evaluate the values (to them) of the respective choice alternatives in each single replication of an experiment, but on average (across replications of the experiment), may reveal a systematic pattern.[1] Alternatively, stochastic choice theories may be interpreted as representing unobserved heterogeneity in a population of agents facing the same choice experiment. The latter type of interpretation is common in the economic literature of discrete choice (see, for example, McFadden, 1981, 2001).

To the best of my knowledge, the first ones to discuss stochastic models for risky choices seem to be Block and Marschak (1960).[2] Subsequently, there have been several attempts of providing axiomatizations for such models, see Fishburn (1978), Gul and Pesendorfer (2006), Blavatskyy (2008, 2011, 2012, 2013), and Dagsvik (2008). This paper discusses different axiomatizations of stochastic models for risky choice proposed by Blavatskyy (2008) and Dagsvik (2008). It is also demonstrated that some of the axioms proposed by Dagsvik (2008) can be weakened without affecting the observable implications.

All the axioms discussed in this paper yield choice models that do not satisfy first order stochastic dominance. In the experimental literature it has been found that individuals rarely violate first order stochastic dominance in choice experiments with money outcomes. Thus, the models discussed here are not appropriate for settings where outcomes can be perfectly rank ordered, such as money. However, I believe that most real life choice settings involve outcomes that cannot easily be rank ordered and thus the axiomatic approaches of Blavatskyy (2008) and Dagsvik (2008) may still be of relevance.

The paper is organized as follows: First, we introduce some basic concepts and definitions in Section 2. In Section 3 we present the axioms of Blavatskyy (2008) and Dagsvik (2008). Section 4 proposes two additional axioms of which one is a weaker version of one of the axioms of Dagsvik (2008), and we discuss some implications from the axioms. Section 5 discusses selected examples and Section 6 summarizes some findings in the experimental literature on risky choices and the issue of statistical testing of measurement axioms.

2. The setting

Let X denote an arbitrary set and $S = S(X)$ the set of all simple probability measures on the algebra of all subsets of X.[3] Let δ_x be

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[1] There are also works where stochasticity arises in other ways, see Marley (1997) and references therein, especially Machina (1985).

[2] Luce (1958) proposed a model for risky choice by introducing random subjective outcome probabilities.

[3] A probability measure g is simple if, for some finite subset, $A \subset X$, $g(A) = 1$.

http://dx.doi.org/10.1016/j.jmateco.2015.06.013
0304-4068/

the degenerate probability measure that assigns all unit mass to the point $x \in X$. Recall that a *preference relation* refers to a binary relation, $\succsim$, on S that is: (i) *complete*, i.e., for all $g_r, g_s \in S$ either $g_r \succsim g_s$ or $g_s \succ g_r$; and (ii) *transitive*, i.e., for all g_r, g_s, g_t, in S, $g_r \succsim g_s$ and $g_s \succsim g_t$ implies $g_r \succsim g_t$. A real-valued function $L(g_s)$ on S is said to *represent* the preference relation if for all g_r, $g_s \in S$, $g_r \succsim g_s$, if and only if $L(g_r) \geq L(g_s)$. Let Ω be a family of finite sets with elements from S. The primitive of choice is a function $P : S \times \Omega \to [0, 1]$, where $P(g_s; B)$, $g_s \in S, B \in \Omega$, represents the probability that g_s is the most preferred lottery among the lotteries in B. Let $P(g_r, g_s)$ be the probability that lottery g_r is chosen over g_s, i.e., $P(g_r, g_s) \equiv P(g_r; \{g_r, g_s\}) = P(g_r \succsim g_s)$. It then follows that $P(g_r, g_s) > P(g_s, g_r)$ if and only if $P(g_r, g_s) > 1/2$. A mixed lottery (compound lottery) $\alpha g_1 + (1 - \alpha) g_2$, is interpreted as a two-stage lottery that yields the lotteries g_1 or g_2 as outcomes with respective probabilities α and $1 - \alpha$ in the first stage and outcome $x_j \in X$ with probability $g_r(x_j)$ in the second stage, given that $g_r(x_j)$ is the outcome in the first stage, $r = 1, 2$. Although compound lotteries are not explicitly used in the main exposition of the paper they are essential for arguments in the proofs given in the Appendix.

The following definition extends the notion of preference to the present context (Blavatskyy, 2008; Dagsvik, 2008).

Definition 1. For $g_r, g_s \in S$, lottery g_r is said to be strictly preferred to g_s in the aggregate sense, if and only if $P(g_r, g_s) > 1/2$. If $P(g_r, g_s) = 1/2$, then g_r is, in the aggregate sense, indifferent to g_s.

Thus, Definition 1 introduces a binary relation, $\succsim$, where $g_r \succ g_s$ means that g_r is strictly preferred to g_s (in the aggregate sense), whereas $g_r \sim g_s$ means that g_r is indifferent to g_s. Thus, $g_r \succsim g_s$ is to be interpreted that g_r is indifferent or preferred to g_r. Note, however, that the relation is not necessarily a *preference relation*. The reason for this is that the binary relation $\succsim$ is *not* necessarily transitive. That is, for $g_1, g_2, g_3 \in S$, the statement that $P(g_1, g_2) \geq 1/2$ and $P(g_2, g_3) \geq 1/2$ imply $P(g_1, g_3) \geq 1/2$ is not necessarily true. For sets, A, B such that $A \subseteq B$, $A, B \in \Omega$, let

$$P(A; B) \equiv \sum_{g_s \in A} P(g_s; B).$$

The interpretation is that $P(A; B)$ is the probability that the agent will choose a lottery within A when B is the choice set.

3. Axioms

In the following we shall apply the notation Axiom B1, B2, etc. for Blavatskyy's axioms and Axiom D1, D2, etc. for Dagsvik's axioms. In Fig. 1 a summary of the axioms is displayed. Before discussing the main Axioms we shall restate a stochastic version of a property called *Axiom of Reduction of Compound Lotteries* (RCL), see Luce and Raiffa (1957).

Axiom RCL. Let $g_1, g_2 \in S$. Then for $\alpha, \beta \in [0, 1]$

$$P(\beta(\alpha g_1 + (1 - \alpha) g_2) + (1 - \beta) g_2, \alpha\beta g_1 + (1 - \alpha\beta) g_2) = 1/2.$$

Thus, the RCL property asserts that the agent is indifferent between the compound lottery $\beta(\alpha g_1 + (1 - \alpha) g_2) + (1 - \beta) g_2$ and the lottery $\alpha\beta g_1 + (1 - \alpha\beta) g_2$. Under the RCL Axiom there will be no loss of generality in assuming that lotteries have only two outcomes since we allow outcomes to be further lotteries. The RCL Axiom is central to theories of risky choice. In this paper it is understood that it holds true even if it is not explicitly stated. Although the RCL Axiom does not appear explicitly in the main text it is used in proofs given in the Appendix.

Axiom D1(i)	⇒	Axiom B2
Axiom D3	⇒	Axiom D1(ii)
Axioms D1(i,iii)	⇒	Axiom B5
Axiom B3	⇒	Axiom D3
Axioms {D1(i,iii), D3, D5}	⇔	Axioms {B1, B2, B3, B4, B5}
Axioms {D1(i,iii), D3, D4}	⇐	Axioms {D3, D5*, D6}
Axioms {D1(i,iii), D3, D4, D5*}	⇔	Axioms {B1, B2, B3, B4, B5}
Axiom D6	⇒	Axiom D1
Axiom D2	⇒	Axiom D6
Axiom D5	⇒	Axiom D4

Fig. 1. Relationship between axioms.

3.1. Blavatskyy's axioms

We shall start with reviewing Blavatskyy's axioms. His first axiom is the so-called Balance condition,

Axiom B1 (*Balance Condition*). Let $g_1, g_2 \in S$. The binary choice probabilities satisfy the Balance condition

$$P(g_1, g_2) + P(g_2, g_1) = 1.$$

Since

$$P(g_1 \succsim g_2) + P(g_2 \succsim g_1) - P(g_1 \sim g_2) = 1$$

where $\sim$ symbolizes indifference we realize that Axiom B1 implies that $P(g_1 \sim g_2) = 0$. In other words, indifference is ruled out. Note also that the Balance condition implies that $P(g, g) = 1/2$.

The next axiom is due to Marschak (1960) who proposed it in the context of choice with perfectly certain outcomes.

Axiom B2 (*Strong Stochastic Transitivity, SST*). For $g_1, g_2, g_3 \in S$, if

$$P(g_1, g_2) > 1/2 \quad \text{and} \quad P(g_2, g_3) > 1/2,$$

then $P(g_1, g_3) \geq \max(P(g_1, g_2), P(g_2, g_3))$.

The following continuity axiom is a stochastic version of the continuity axiom in the expected utility theory, originally due to Herrnstein and Milnor (1953).

Axiom B3 (*Continuity*). For $g_1, g_2, g_3 \in S$, the sets

$$\{\alpha \in [0, 1] : P(\alpha g_1 + (1 - \alpha) g_3, g_2) \geq 1/2\} \quad \text{and}$$
$$\{\alpha \in [0, 1] : P(\alpha g_1 + (1 - \alpha) g_3, g_2) \leq 1/2\}$$

are closed.

To extend the so-called independence axiom in the expected utility theory Blavatskyy proposes the following axiom.