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An FFT-based signal identification approach for obtaining the propagation constants of the leaky modes in layered media

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Abstract

We propose to find the propagation constants of modes in layered media by means of signal identification methods. To this effect we employ Cauchy's theorem, conformal mapping and Fast Fourier Transform (FFT) techniques to generate relevant Hankel moments, afterwards to be processed with selected signal identification algorithms. The method, terminated by a few Newton steps, provides a batch of highly accurate roots in appropriate disks or half-disks.

Keywords: Electromagnetic propagation in stratified media; Leaky waves; Signal resolution; Fast Fourier transforms; Zeros

1. Introduction

Leaky waves, which are found as complex solutions of the dispersion relation for dielectric waveguides, play an important role in many electromagnetic (EM) analysis tools [1–3] as well as in practical devices such as leaky wave antennas [4]. Locating the complex zeros, however, can be quite cumbersome, especially when the dielectric substrate consists of a number of different dielectric layers. In this paper, we propose techniques based on signal identification algorithms to efficiently and accurately locate the complex zeros of dispersion relations for layered media. The techniques can be applied for layered substrates consisting of an arbitrary number of layers. The top and bottom substrate can either be open or, for numerical reasons, terminated by a perfectly matched layer (PML) [5–7].

Signal identification of exponential sum models (ESM) is a frequently occurring and recurrent topic in signal processing. The reason for this is that many physical signals, from time series in medicine and economics to spectral analysis in astronomy and sonar applications [8,9] can be expressed as sums of damped exponentials. Moreover, ESM has also been used in EM analysis techniques, e.g. in the complex image method for determining the Green's function in layered media. More mathematically speaking, it can be proved that large classes of signals can be expressed as infinite sums of exponentials, due to their L_p completeness over selected intervals [10,11]. In addition, modelling by exponential sums is inherent in linear systems theory [12] and its underlying Hankel matrix framework [13,14]. Lastly and rather unexpectedly, exponential signal identification has recently been used for reconstructing polygonal shapes from geometrical moments [15].

In this paper, we propose to find the roots of transcendental equations by means of signal identification techniques. As a generalization of methods in [16,17], we employ Cauchy's theorem, conformal mapping and FFT techniques to generate the relevant Hankel moments, which are afterwards processed with selected signal identification algorithms. Our choice (not exhaustive) is one of the four following algorithms: the Pencil-of-Function method [18], the SVD rank-based method [9], the Prony–Burrus–Parks method [19,20] and a Neville-type interpolation method [21]. Applied to finding the propagation constants of the

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leaky modes in stratified media, which correspond to the complex zeros of transcendental dispersion relations, the method, terminated by a few Newton steps, provides a batch of highly accurate roots in selected disks or half-disks.

2. Roots of transcendental equations

It is well-known [16,17] that, given a function $F(z)$ which is analytic in a simply connected open domain Ω bounded by a simple Jordan curve Γ, except for a simple pole at p_0, we can recover this pole by means of the formula

$$p_0 = \frac{\oint_\Gamma zF(z)\,\mathrm{d}z}{\oint_\Gamma F(z)\,\mathrm{d}z}. \tag{1}$$

This follows at once from Cauchy's theorem. For an analytic function $f(z)$ exhibiting a single simple zero z_0 in Ω we can take $F(z) = 1/f(z)$ in order to find z_0 by formula (1). Of course, one must be sure that only one zero is present in Ω. This can be tested by making use of the principle of the argument [22], which states that for an analytic function $f(z)$ with m zeros inside Ω we can write

$$m = \frac{1}{2\pi i}\oint_\Gamma \frac{f'(z)}{f(z)}\,\mathrm{d}z. \tag{2}$$

When $m=1$, formula (1) will yield the unique zero. In order to generalize the approach to more than one zero we need the following [17].

Theorem 1. *Let the analytic function $f(z)$ have exactly m simple zeros $z_1, z_2, \ldots, z_m$ in Ω and let $g(z)$ be analytic such that $g(z_k) \neq 0$ for $k = 1, 2, \ldots, m$. Then there exist non-vanishing coefficients $\tilde{d}_k$, $k = 1, 2, \ldots, m$ such that the Hankel moments*

$$h_n = \frac{1}{2\pi i}\oint_\Gamma \frac{g(z)}{f(z)} z^n\,\mathrm{d}z = \sum_{k=1}^{m} \tilde{d}_k z_k^n, \quad n = 0, 1, 2, \ldots . \tag{3}$$

Proof. The function $f(z)$ can be written as

$$f(z) = \frac{\prod_{k=1}^{m}(z - z_k)}{r(z)}, \tag{4}$$

where $r(z)$ is analytic in Ω with $r(z) \neq 0$. Since

$$\frac{1}{\prod_{k=1}^{m}(z - z_k)} = \sum_{k=1}^{m} \frac{c_k}{z - z_k} \tag{5}$$

with

$$c_k = \frac{1}{\prod_{l\neq k}(z_k - z_l)},$$

application of Cauchy's theorem yields

$$h_n = \frac{1}{2\pi i}\oint_\Gamma \frac{g(z)}{f(z)} z^n\,\mathrm{d}z = \sum_{k=1}^{m} c_k g(z_k) r(z_k) z_k^n \tag{6}$$

with $n = 0, 1, 2, \ldots$. This completes the proof. □

Note that a straightforward choice for the function $g(z)$ is the constant function $g(z) = 1$. The other straightforward choice $g(z) = f'(z)$, with $n = 0$, will be employed mainly to determine the number of zeros m.

We will restrict ourselves to zeros in $\mathscr{D}$, the open unit disk for the following reason: consider the conformal mapping $\eta(u)$ from $\mathscr{D}$ onto Ω. This mapping always exists, by virtue of the Riemann mapping theorem [23]. Then the zeros of $f(z)$ in Ω correspond with the zeros of $f(\eta(u))$ in $\mathscr{D}$. Taking $g(u) = 1$, the Hankel moments h_n can be written as

$$h_n = \frac{1}{2\pi}\int_0^{2\pi} \frac{\mathrm{e}^{i(n+1)\theta}}{f(\eta(\mathrm{e}^{i\theta}))}\,\mathrm{d}\theta = \sum_{k=1}^{m} d_k u_k^n \tag{7}$$

with $n = 0, 1, 2, \ldots$. Opting for the $(N+1)$-point closed trapezoidal quadrature rule we obtain

$$h_n = \frac{1}{N}\sum_{k=0}^{N-1} \frac{\mathrm{e}^{2\pi i(n+1)k/N}}{f(\eta(\mathrm{e}^{2\pi ik/N}))} + \varepsilon_n, \tag{8}$$

where ε_n is the error associated with the quadrature rule. Assuming that the error terms are sufficiently small—see the appendix for some pertinent error bounds—and choosing N as a power of two, (8) is most effectively calculated by means of an FFT. After obtaining the Hankel moments, and processing them with one or more of the signal identification algorithms of the next section, we finally obtain the unit disk zeros u_k and the Ω-domain zeros by $z_k = \eta(u_k)$.

3. Signal identification algorithms

Given the Hankel moments

$$h_n = \sum_{k=1}^{m} d_k u_k^n, \quad n = 0, 1, 2, \ldots, \tag{9}$$

the zeros u_k are recovered by judiciously processing the moments h_n by means of one of the following four algorithms. Note that there exist other algorithms, such as the ones in [13,24], but we have to be restrictive somehow, and therefore we limit ourselves to the non-exhaustive but representative list below.

3.1. Algorithm 1: Pencil-of-Function method

Consider the $m \times m$ Hankel matrix

$$H_m = \begin{pmatrix} h_0 & \cdots & h_{m-1} \\ \vdots & \cdots & \vdots \\ h_{m-1} & \cdots & h_{2m-2} \end{pmatrix} \tag{10}$$