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An interim core for normal form games and exchange economies with incomplete information[☆]

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ABSTRACT

We consider the *interim* core of normal form cooperative games and exchange economies with incomplete information based on the partition model. We develop a solution concept that we can situate roughly between Wilson's coarse core and Yannelis's private core. We investigate the *interim* negotiation of contracts and address the two situations of contract delivery: *interim* and *ex post*. Our solution differs from Wilson's concept because the measurability of strategies in our solution is postponed until the consumption date (assumed with respect to the information that will be known by the players at the consumption date). For *interim* consumption, our concept differs from Yannelis's private core because players can negotiate conditional on proper common knowledge events in our solution, which strengthens the *interim* aspect of the game, as we will illustrate with examples.

1. Introduction

We define and investigate an *interim* core concept for normal form games with incomplete information. We focus on the α-core of normal form games (Aumann, 1961; Scarf, 1971; Kajii, 1992) and the core of exchange economies as initiated by Radner (1968). The incomplete information aspect is modeled using Wilson's (Wilson, 1978) partition model. Specifically, we associate to each player a σ-field representing the events that the particular player can discern.

Wilson (1978) defined two concepts of the core that targeted two extreme situations: in the *coarse core* concept, agents are not permitted to share their information, whereas in the *fine core* concept, agents share all their information within coalitions. Wilson (1978) obtained the non-emptiness of the coarse core by endowing the grand coalition with the particular power of sharing all its agents' information, while proper coalitions do not share any information and negotiate only over "common knowledge" events. The incoherence of this particular construction has been criticized in the literature, which has spawned alternative models.

Several interesting studies have been undertaken based on Wilson's solution concepts. Allen (1996) provided an overview of the basic literature concerning exchange economies and cooperative games. Yannelis (1991) defined the private core, in which agents do not share their private information. Part of the incomplete information aspect of the private core and other types of models is represented by assuming that agents can only envision strategies that are measurable with respect to the σ-fields of events that they can discern, which amounts to assuming that contract delivery is situated at the *interim* stage. Koutsougeras and Yannelis (1993) investigated the incentive compatibility of core concepts under incomplete information. The incentive compatibility problem arises when the prevailing state of nature is not publicly known prior to consumption or contract delivery. The investigation of the private core was further undertaken by Glycopantis et al. (2001) and Allen and Yannelis (2001). Page (1997) formulated a common treatment and proved the non-emptiness of the core for a unified model that can be reduced to some version of Wilson's coarse and fine cores[1] and Yannelis's private core as a function of information-sharing rule. Serfes (2001) extends the private core to dynamic economies and defines a non-myopic core. Additional information on this subject, incentive problems and a cogent mathematical formulation of the problem of information sharing is provided in Allen (2006). For a review of the alternative approach (in incomplete information games), Harsanyi's model Harsanyi, 1967–1968, we refer to the survey of Forges et al. (2002).

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[1] Indeed, Page (1997) used *ex ante* utilities, whereas in each of Wilson's coarse and fine cores, agents evaluate allocations based on possibly proper events that are smaller than the entire universe.

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In this paper, we formulate an *interim* core concept in which players negotiate with *interim* utilities (conditional expectations) and can object to a status-quo allocation (strategy) for common knowledge events. The difference between our concept and Wilson's concepts is that the measurability of strategies is not related to the information available at the negotiation date but to the information that will be known by the players at the consumption date. This modeling idea is more natural and lends consistency to the resulting concepts. Moreover, it solves the incoherence problem in Wilson's construction. From this perspective, players faces all circumstances that will be known just before contract delivery. In other words, they can envision strategies based on the information that will be received before consumption, and are not restricted to using strategies conceivable under the information known during the negotiation stage. Negotiation is conducted using the available information, but strategies can be based on all information that will be known – even future information – before consumption.

If consumption is situated at the *interim* stage (no additional information between negotiation and consumption), then our concept is close to Yannelis's private core, except that the negotiation is conducted on proper common knowledge events. Compared with the private core,[2] this strengthens the *interim* aspect of the model, as we will show with examples in Section 3.2. We address two situations, *i.e.*, whether the prevailing state of nature is revealed at the consumption date or whether it is not. The α-core of a normal form game and the core of an exchange economy are addressed in each case. Note that Askoura et al. (2013) and Noguchi (2014) provide an *ex ante* formulation of the α-core, under Harsanyi's model.

This paper is divided into two main parts: Section 2 is devoted to *ex post* contract delivery and contains most of the notation and technical tools that we use. Section 3 is devoted to *interim* contract delivery and concludes with a subsection comparing our concept to the private core. Section 4 contains a closing comment on information sharing.

2. *Ex post* contract delivery

In an incomplete information model, two important aspects must be considered: (a) at what stage are negotiations made, *ex ante* or *interim*?[3] and (b) at what stage is contract delivery made, *ex ante*, *interim* or *ex post*? For (a), this paper discusses the *interim case*. For (b), two consumption dates, *interim* and *ex post*, are addressed. The *interim* consumption is technically accounted for by the assumption of the measurability of players' strategies, which can be roughly addressed by acknowledging that players can only envision strategies that are measurable with respect to their respective σ-fields.[4] Then, this assumption is related to conditions under which the game takes place. For instance, it is unnecessary if the prevailing state (or, generally, a finer σ-field than all players' information σ-fields) is revealed before contract delivery, and it is necessary otherwise. In other words, if contract delivery is made at the *ex post* stage, the measurability condition can be relaxed, such as is the case in Volij (2000), in which the agents' utilities are updated again following the information that agents possess at the negotiation step. Volij (2000) obtained an intermediate core concept between the coarse and the fine core of Wilson. In Okada (2012), a new type of core (informational core) was introduced in which the measurability assumption is no longer required. In Kobayashi (1980), measurability was assumed with respect to a σ-field that will be revealed before contract delivery and a conditional core was introduced. In this section, consumption is organized *ex-post*.

2.1. Conceptual aspects—normal form games

Let $(\Omega, \mathcal{F}, \mu)$ be a probability space. Ω represents the set of states of nature. The probability μ is a common objective prior. In the sequel, two events with a μ-null symmetric difference will be confused. $N = \{1, \dots, n\}$ is the set of players. If $S \subset N$ is a coalition, then we denote by $-S$ the coalition of the remaining players. The action space of player i is denoted A_i. It is assumed to be a compact convex subset of a (separable) Banach space X_i. The σ-algebra $\mathfrak{B}(A_i)$ stands for the Borel σ-field of A_i.

Set $A = \prod_{i \in N} A_i$. The information of each player i is represented by a sub-σ-algebra $\mathcal{F}_i$ of $\mathcal{F}$. The elements of $\mathcal{F}_i$ represent the events that player i can discern. In other words, for every $E \in \mathcal{F}_i$, player i knows whether the prevailing state is in E or in its complement, which is denoted $\complement E$. Wilson (1978) defined the fine core to be the core concept corresponding to the situation in which agents within a coalition S pool all their information. Then, they can discern all events in the coarsest sub-σ-algebra generated by $\bigcup_{i \in S} \mathcal{F}_i$, which is denoted $\bigvee_{i \in S} \mathcal{F}_i$. Analogously, Wilson's coarse core is that in which agents do not reveal their information within coalitions or use common knowledge events or events contained therein for a coalition S in the field $\bigcap_{i \in S} \mathcal{F}_i$, which is denoted $\bigwedge_{i \in S} \mathcal{F}_i$.

(R1) Assume that there is a finite partition of Ω generating the σ-algebra $\bigvee_{i \in N} \mathcal{F}_i$; then, every sub-$\sigma$-field $\mathcal{F}_i$ is generated by a partition of Ω. Denote by $\mathcal{P}$ the coarsest partition generating the field $\bigvee_{i \in N} \mathcal{F}_i$. Denote by $\mathcal{P}_i$ the coarsest partition of Ω generating $\mathcal{F}_i$. We can assume that $\mu(K) > 0$ for all $K \in \mathcal{P}_i$. The elements of $\mathcal{P}_i$ are the finest events that can be discerned by player i without sharing information with other players. Observe that for every $K \in \mathcal{P}$, there is a unique $K_i \in \mathcal{P}_i$ such that $K \subset K_i$.

Each agent $i \in N$ is assumed to know, before the negotiation date, the smallest (finest) event (an element of $\mathcal{P}_i$) in his field $\mathcal{F}_i$ containing the realized state of nature.[5]

For each player i, associate a utility function

$$u_i : A \times \Omega \to \mathbb{R}_+.$$

In the sequel, we assume that

- the information fields $\mathcal{F}_i$, $i \in I$, and the other components of the game are publicly known.

For a player, knowing the whole structure of the information, that is, all the fields $\mathcal{F}_i$, $i \in N$, does not provide him with more information if he is only assumed to discern, in his own field, whether any event contains the prevailing state.

Player i's strategy,[6] x_i, is a function from Ω into his space of actions.[7] The measurability condition requires that x_i be measurable with respect to the σ-field containing the events that player i can discern. For instance, this function must be $\mathcal{F}_i$-measurable in the absence of information sharing or, for example, $\bigvee_{j \in S} \mathcal{F}_j$-measurable if i belongs to the coalition S and S shares all its members' information. We consider the measurability condition by distinguishing two situations:

2 This comment may be put in some perspective by the version of the private core defined in Hahn and Yannelis (2001) and in De Castro et al. (2011) as a weak *interim* private core. See point 2 in Section 3.2 for a discussion of this concept: in particular, it may fail to be nonempty under usual assumptions.

3 The *ex post* situation corresponds to complete information.

4 Alternatively, new σ-fields are generally defined by the information sharing rule.

5 This is only true in the case in which $\mathcal{F}_i$ is generated by a partition of Ω, as assumed above; otherwise, at this stage, player i can only know whether the realized state is in E or in $\complement E$, for all $E \in \mathcal{F}_i$.

6 Allocation in the case of exchange economies.

7 Consumption set in the case of an exchange economy.