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Ambiguity on the insurer's side: The demand for insurance

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ABSTRACT

Empirical evidence suggests that ambiguity is prevalent in insurance pricing and underwriting, and that often insurers tend to exhibit more ambiguity than the insured individuals (e.g., Hogarth and Kunreuther, 1989). Motivated by these findings, we consider a problem of demand for insurance indemnity schedules, where the insurer has ambiguous beliefs about the realizations of the insurable loss, whereas the insured is an expected-utility maximizer. We show that if the ambiguous beliefs of the insurer satisfy a property of compatibility with the non-ambiguous beliefs of the insured, then optimal indemnity schedules exist and are monotonic. By virtue of monotonicity, no *ex-post* moral hazard issues arise at our solutions (e.g., Huberman et al., 1983). In addition, in the case where the insurer is either ambiguity-seeking or ambiguity-averse, we show that the problem of determining the optimal indemnity schedule reduces to that of solving an auxiliary problem that is simpler than the original one in that it does not involve ambiguity. Finally, under additional assumptions, we give an explicit characterization of the optimal indemnity schedule for the insured, and we show how our results naturally extend the classical result of Arrow (1971) on the optimality of the deductible indemnity schedule.

1. Introduction

The classical formulation of the problem of demand for insurance indemnity schedules is due to Arrow (1971): a risk-averse Expected-Utility (EU) maximizing individual faces an insurable random loss X, against which he seeks an insurance coverage; and, a risk-neutral EU-maximizing insurer is willing to insure this individual against the realizations of the random loss, in return for an upfront premium payment. The insured seeks an indemnity schedule that maximizes his expected utility of final wealth, subject to the given premium Π determined by the insurer. Arrow's (1971) classical theorem states that in this case, the optimal insurance indemnity schedule takes the form of full insurance above a constant positive deductible. That is, there exists a constant $d \geqslant 0$ such that the optimal insurance indemnity schedule is of the form

$$Y^* = \max\Big(0, X - d\Big).$$

This is a pure risk-sharing result: both parties have the same probabilistic beliefs, and the need for insurance is a consequence only of their different attitudes toward risk. Ghossoub (2013) extended Arrow's result to the case of heterogeneous beliefs. Ghossoub's (2013) result and *a fortiori* Arrow's (1971) result apply, however, only to those situations where both parties have rich information about the relevant uncertainty, so as to be able to reduce that uncertainty to risk and form a probabilistic assessment. In contrast, empirical evidence suggests that ambiguity (as opposed to risk) is prevalent in insurance pricing and underwriting, and that often insurers tend to exhibit more ambiguity than the insured individuals (e.g., Hogarth and Kunreuther, 1989). Motivated by these findings, we re-examine the classical insurance demand problem of Arrow (1971) in a setting where the insurer has ambiguous beliefs (in the sense of Schmeidler, 1989) about the realizations of the insurable loss, whereas the insured is an EU-maximizer.

Formally, we examine a problem similar to that of Arrow (1971), with the sole difference that the beliefs of the insurer are represented by a capacity (Appendix A, Definition A.1) rather than a probability measure. Our results are as follows. First, we present a general result (Theorem 4.6), which states that if the parties' beliefs satisfy a certain compatibility condition (Definition 4.5), then optimal indemnity schedules exist and are monotonic. Here,

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monotonicity means that the optimal indemnity schedule is a nondecreasing function of the realizations of the insurable loss random variable. As it is well-known, this property rules out *ex post* moral hazard issues that could arise from the possibility that the insurer could misreport the actual amount of loss suffered (Huberman et al., 1983). This result complements a similar result that we obtained in Amarante et al. (2011) for a slightly different setting (which involves some minor technical differences).

We then consider the case where the insurer is either ambiguity-seeking or ambiguity-averse in the sense of Schmeidler (1989). We show that in both cases, an optimal indemnity schedule can be replicated by an optimal indemnity obtained from an insurance problem in which both the insured and the insurer are EU-maximizers, but have different beliefs about the realizations of the insurable random loss (Propositions 5.1 and 6.1). Such problems have been recently studied by Ghossoub (2013).

Finally, under additional assumptions, we obtain an explicit characterization of the optimal indemnity schedule as a function of the underlying data. In the case of an ambiguity-seeking insurer whose capacity is a distortion of the probability measure of the insured, we show that the optimal indemnity schedule takes the form

$$Y^* = \min\left[X, \max\Big(0, X - d\,(T)\Big)\right],$$

where T is the concave probability distortion function of the insurer (see Appendix A), and $d\,(T)$ is a state-contingent deductible that depends on the state of the world only through the function T (Theorem 5.4). In the case of an ambiguity-averse insurer whose capacity has a core (Appendix A, Definition A.2) consisting of probability measures with the monotone likelihood ratio (MLR) property, we show that the optimal indemnity schedule is a state-contingent deductible of the form

$$Y^* = \min\left[X, \max\Big(0, X - d\,(LR)\Big)\right],$$

where LR denotes a function of the likelihood ratios of the probabilities in the core of the supermodular capacity over the probability of the insured (Corollary 6.3). In both cases, we determine the state-contingent deductible d explicitly. Arrow's solution obtains as a limit case from both settings: when the distortion function T becomes the identity function in the ambiguity-seeking case and when the core collapses to the probability measure of the insured in the ambiguity-averse case.

Related literature

The literature on ambiguity in insurance design can be split into two main streams: (i) ambiguity on only one side of the insurance problem, and (ii) ambiguity on both sides. In the former category, all of the work that has been done has invariably assumed that the ambiguity is on the side of the insured. As such, it is very different from what we do in this paper. For instance, Alary et al. (2013) consider an insured who is ambiguity-averse in the sense of Klibanoff et al. (2005), and assume that the ambiguity is concentrated only in the probability that a loss occurs. Conditional on a loss occurring, the distribution of the loss severity is unambiguous. Under these assumptions, they show that the optimal indemnity is a straight deductible. Gollier (2012) also focuses on the case of an insured who is ambiguity-averse in the sense of Klibanoff et al. (2005). He shows that if the collection of priors can be ordered according to the MLR property, then the optimal indemnity schedule contains a disappearing deductible. Jeleva (2000) considers an insurance model in which the insurer is Choquet-Expected Utility (CEU) maximizer (Schmeidler, 1989). She specifies *ex ante* that the insurance contract is of the co-insurance type, and she then examines the optimal co-insurance factor. Young (1999) and Bernard et al. (2015) examine the case where the insured is a Rank-Dependent Expected-Utility maximizer (Quiggin, 1982; Yaari, 1987). Doherty and Eeckhoudt (1995) study the optimal level of deductible under Yaari's Dual Theory (Yaari, 1987). Karni (1992) and Machina (1995) consider a setting where the preferences of the insured have a non-EU representation that satisfies certain differentiability criteria. The former shows that a deductible indemnity schedule is optimal; whereas he latter examines the optimal level of co-insurance and optimal level of deductible. Schlesinger (1997) examines the optimal co-insurance level in a situation where the preferences of the insured are not necessarily EU preferences, but they are risk-averse in the sense of disliking men-preserving increases in risk.

In the second stream of the literature on ambiguity in insurance design, which contemplates ambiguity on both sides, Carlier et al. (2003) consider the case in which both parties' beliefs are epsilon-contaminations of a given prior, and they show that the optimal indemnity contains a deductible for high values of the loss. Anwar and Zheng (2012) allow for both two-sided ambiguity and belief heterogeneity but restrict to a model with only two states of the world. As such, the scope of their inquiry is limited because, in general, financial and insurance risks are not binary risks (as they would necessarily be in a two-state model). Moreover, the shape of an optimal indemnity schedule cannot be determined in a two-state model where the loss X can take only two values[1]: L with probability p, and 0 with probability $1 - p$.

More general problems that are directly relevant to the insurance problem considered here have been examined by Carlier and Dana (2002, 2003, 2008) and Chateauneuf et al. (2000). However, none of these studies provide a full characterization of the optimal insurance indemnity schedule, which is one of the main goals of the present paper.

2. Setup

Let S denote the set of states of the world, and suppose that $\mathcal{G}$ is a σ-algebra of subsets of S, called events. Denote by $B\,(\mathcal{G})$ the linear space of all bounded, $\mathbb{R}$-valued and $\mathcal{G}$-measurable functions on $(S, \mathcal{G})$, and denote by $B^+\,(\mathcal{G})$ the collection of all $\mathbb{R}^+$-valued elements of $B\,(\mathcal{G})$. Any $f \in B\,(\mathcal{G})$ is bounded, and we define its supnorm by $\|f\|_{sup} := \sup\{|f\,(s)| : s \in S\} < +\infty$.

Suppose that an individual has initial wealth W_0 and is facing an insurable random loss X, against which he seeks insurance. This random loss is a given element of $B^+\,(\mathcal{G})$ with closed range $X\,(S) = [0, M]$, where $M := \|X\|_{sup} < +\infty$. Denote by Σ the σ-algebra $\sigma\{X\}$ of subsets of S generated by X. Then by Doob's measurability theorem (Aliprantis and Border, 2006, Theorem 4.41), for any $Y \in B\,(\Sigma)$ there exists a Borel-measurable map $I : \mathbb{R} \to \mathbb{R}$ such that $Y = I \circ X$. Denote by $B^+\,(\Sigma)$ the cone of nonnegative elements of $B\,(\Sigma)$. Let P be a probability measure on (S, Σ). We will make the following assumption all throughout.

Assumption 2.1. The random loss X is a continuous random variable[2] on the probability space (S, Σ, P). That is, the Borel probability measure $P \circ X^{-1}$ is nonatomic.[3]

[1] At least if one imposes, as it is customary, the constraint that the indemnity be non-negative and not larger than the loss itself.

[2] This is a standard assumption, and it holds in many instances, such as when it is assumed that a probability density function for X exists.

[3] A finite nonnegative measure η on a measurable space $(\Omega, \mathcal{A})$ is said to be *nonatomic* if for any $A \in \mathcal{A}$ with $\eta\,(A) > 0$, there is some $B \in \mathcal{A}$ such that $B \subsetneq A$ and $0 < \eta\,(B) < \eta\,(A)$.