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Complexity results for the weak axiom of revealed preference for collective consumption models

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ABSTRACT

The purpose of this paper is to establish the complexity of alternative versions of the weak axiom of revealed preference (WARP) for collective consumption models. In contrast to the unitary consumption model, these collective models explicitly take the multi-member nature of the household into account. We consider the three collective settings that are most often considered in the literature. We start with the private setting in which all goods are privately consumed by the household members. Next, we consider the public setting in which all goods are publicly consumed inside the household. Finally, we also consider the general setting where no information on the (private or public) nature of goods consumed in the household is available. We prove that the collective version of WARP is NP-hard to test for both the private and public settings. Surprisingly, we also find for the general setting that the collective version of WARP is easy to test for two-member households.

1. Introduction

Modeling and analyzing household consumption behavior is a fundamental research topic in microeconomics. For a long time, the standard model in empirical consumption analysis was the so-called *unitary* model, which treats the household as a single decision making unit. However, by now it is well established that Chiappori (1988)'s *collective* model of household consumption is both conceptually and empirically a more attractive alternative for analyzing the consumption behavior of multi-member households (see, for example, Vermeulen, 2002 for an overview of the relevant literature). This collective model assumes that the different household members are endowed with individual preferences defined over privately and publicly consumed goods (inside the household). These members then enter into a decision process of which the outcome is assumed to obtain a Pareto optimal allocation (of the aggregate household budget).

In the tradition of Afriat (1967) and Varian (1982), we are interested in the revealed preference characterization of collective models. Such a revealed preference characterization does not rely on any functional specification regarding the household consumption process, and starts directly from the observed finite set of prices and quantities. Varian (1982) introduced the revealed preference axioms that summarize the empirical implications of theoretical consumption models for single-member households. Basically, consistency with the unitary model requires the observed consumption data to obey the *strong axiom of revealed preference* (SARP). More recently, Cherchye et al. (2011) provided a revealed preference characterization of collective models for multi-member households, which implies a multi-member version of SARP.[1]

To date, this collective extension of SARP has received extensive analysis. Most notably, it has been shown that testing the SARP conditions for collective models is NP-complete, even for households with only two members (Deb, 2010; Talla Nobibon et al.,

[1] See also Peters and Wakker (1994), Varian (2006), Cherchye et al. (2007), Cherchye et al. (2010) and Cherchye et al. (2013) for more discussion. To be precise, Cherchye et al. (2011) actually characterized the collective model in terms of the *generalized axiom of revealed preference* (GARP) rather than SARP. But their results are easily translated towards *SARP*. See, for example, Varian (1982) for a detailed discussion on the subtle difference between GARP and SARP.

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2012; Talla Nobibon and Spieksma, 2010). Importantly, this contrasts with complexity results for the unitary model, for which the SARP conditions can be tested in polynomial time (Piaw and Vohra, 2003; Talla Nobibon et al., 2014; Varian, 1982).

In this paper, we complement these existing results by considering the collective version of the so-called *weak axiom of revealed preference* (WARP). Basically, WARP coincides with SARP except from the fact that it does not require revealed preferences to be transitive. Thus, in general, WARP and SARP are not equivalent to each other. However, it turns out that in practical applications they often have identical empirical implications, i.e. most data that satisfy WARP also satisfy SARP. Putting it differently, in empirical work transitivity usually plays little role when testing data consistency with revealed preference axioms. This last observation is an important one in view of practical tests of the collective model (SARP) restrictions, as transitivity restrictions usually occupy a lot of computation time.

This also directly motivates the purpose of the current paper, which focuses on the computational complexity of the collective WARP conditions. Tools of computational complexity are increasingly used to investigate computational properties of various problems in economic behavior; we restrict ourselves here to mentioning work in goodness-of-fit models (Echenique et al., 2011; Smeulders et al., 2014) and rationalizing choice behavior (Apesteguia and Ballester, 2010; Demuynck, 2011). Essentially, we will evaluate whether the computational hardness of the collective revealed preference conditions can be mitigated by dropping the transitivity requirement. In particular, our following analysis will consider the WARP characterization of three collective consumption settings: (i) the *private* setting where all goods are consumed privately without externalities, (ii) the *public* setting, where all goods are publicly consumed inside the household, and (iii) a *general* setting where no information on the (private or public nature) of the goods is available.

Our main findings can be summarized as follows. A first "negative" conclusion will be that testing the collective WARP conditions is computationally difficult (i.e. NP-complete) for the private and public settings. In these cases, dropping transitivity does not solve the hardness problem associated with the collective SARP conditions. However, as a second "positive" conclusion, we also show that testing collective WARP for two members is computationally easy for the general setting. Here, we can effectively test consistency with the collective consumption model in an efficient way (i.e. in polynomial time) if we omit transitivity. (As we will indicate, for this general setting the complexity in the case of three or more members remains an open question.)

The remainder of the paper unfolds as follows. Section 2 presents our basic set-up. Sections 3–5 contain our main complexity results (for, respectively, the private, public and general settings). Section 6 concludes.

2. Set-up

To set the stage, we will start by fixing our basic notation. Next, we will introduce the WARP conditions that apply to single-member households (which are essentially the conditions that apply to the unitary consumption model). The following sections will be more specific on the collective models (and corresponding WARP conditions) that form the central focus of our analysis.

2.1. Notation

We consider multi-member households that take consumption decisions over m commodities (or goods). These goods can be consumed either privately (with or without externalities) or publicly. More precisely, *private* consumption of a good means that the consumption by one household member affects the supply available for the other household members (e.g. drinking water can only be consumed privately). Next, consumption *externalities* refer to the fact that one household member gets utility from another household member's private consumption (e.g. a wife enjoys her husbands nice clothes). Finally, *public* consumption of a good means that consumption of that good by one household member does not affect the supply available for the other household members, and no one can be excluded from consuming the good (e.g. the rent of a shared house represents public consumption).

The collective models of household consumption explicitly recognize the individual preferences of the household members. These preferences may depend on the private quantities (with or without externalities), the public quantities, or both. Throughout, we assume that preferences of the household members can be represented by a well-behaved (i.e. continuous, positive monotonic and concave) utility function. The following sections will define explicit specifications of these member-specific utility functions for alternative collective consumption models.

We assume a setting in which the empirical analyst observes n household decisions resulting in consumption quantity bundles $q_t := (q_{t,1}, \ldots, q_{t,m}) \in \mathbb{R}^m_+$, with corresponding prices $p_t := (p_{t,1}, \ldots, p_{t,m}) \in \mathbb{R}^m_{++}$, $t = 1, \ldots, n$. The component $q_{t,i}$ (respectively $p_{t,i}$), for $i = 1, \ldots, m$, corresponds to the quantity of good i bought by the household (respectively, the unit price of good i) at the time of observation t. Note that the scalar product $p'q$ represents the total outlay for bundle $q \in \mathbb{R}^m_+$ at the prices $p \in \mathbb{R}^m_{++}$. For ease of notation, we will write this scalar product simply as pq. We denote the set of observations by $S := \{(p_t, q_t) : t \in N\}$, where $N := \{1, \ldots, n\}$, and we refer to S as the *dataset*. For ease of exposition, throughout this paper, we use $t \in N$ to refer to the observation (p_t, q_t).

2.2. WARP for single-member households

Samuelson (1938) originally introduced the WARP condition for single-member households. It defines a necessary requirement for the existence of a single well-behaved utility function that is consistent with the observed dataset $S := \{(p_t, q_t) : t \in N\}$. More precisely, there can only exist a well-behaved utility function that is maximized by each observed bundle q_t subject to the corresponding budget constraint (defined by the prices p_t and the budget $p_t q_t$), if the dataset S is consistent with WARP. As indicated in the Introduction, the necessary WARP requirement differs from the necessary and sufficient SARP requirement in that it does not require (revealed) preferences to be transitive.

The formal definition of WARP is as follows.

Definition 1. Let $S := \{(p_t, q_t) : t \in N\}$ be an observed dataset.

1. Bundle q_s is *directly revealed preferred* over bundle q_t if and only if $p_s q_s \geq p_s q_t$.
2. S satisfies WARP if and only if, for all observations s, $t \in N$, when $q_s \neq q_t$ and q_s is directly preferred over q_t, then $p_t q_t < p_t q_s$.

In words, the bundle q_s is directly revealed preferred over the bundle q_t if the bundle q_s was chosen, while the bundle q_t was also affordable (at the prices p_s). Next, if a bundle q_s is directly revealed preferred over q_t, then it cannot be that q_t is also directly revealed preferred over q_s (unless both bundles are identical).

In the following sections, we will extend this single-member WARP requirement to alternative collective settings. Collective consumption models pertain to multi-member households, in which each member has his or her own well-behaved utility function. Therefore, our consistency conditions for collective models will basically impose WARP requirements for the (multiple) individual household members. The specific form of these WARP requirements will depend on the collective model at hand.