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Perfect simulation for models of industry dynamics ☆

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ABSTRACT

In this paper we introduce a technique for perfect simulation from the stationary distribution of a standard model of industry dynamics. The method can be adapted to other, possibly non-monotone, regenerative processes found in industrial organization and other fields of economics. The algorithm we propose is a version of coupling from the past. It is straightforward to implement and exploit the regenerative property of the process in order to achieve rapid coupling.

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1. Introduction

The dynamics of entry and exit by firms play an essential role in economic theory as well as in real life. Productive new entrants replace unproductive incumbents, rejuvenating overall economic activity. There is a large and growing literature on this economic mechanism (see, e.g., Hopenhayn and Rogerson, 1993, Cooley and Quadrini, 2001 or Melitz, 2003), and much of this literature builds upon the model of entry and exit studied by Hopenhayn (1992). The stationary distribution of entry–exit models of the type studied by Hopenhayn represents a cross-sectional distribution of firms that is both consistent with the definition of equilibrium at any point in time and also invariant over time. For typical parameter values the stationary distribution is uniquely defined but has no analytical solution.[1]

Simulation is a useful option for computing the cross-sectional distribution for a given exit policy, since it is not difficult to write down an ergodic Markov process such that its stationary distribution coincides with the cross-sectional stationary distribution. This permits approximate sampling from the stationary distribution simply by running the process from an arbitrary initial value until it is judged to have "nearly" converged. Simulating until the distribution of the state is approximately stationary is referred to as "burn-in". Unfortunately the length of burn-in required is often the subject of guesswork and heuristics.[2] Moreover, regardless of how much burn-in is performed, the resulting sample is never exactly stationary, and the size of the error is once again unknown.[3]

In this paper we show that these problems can be overcome. By using a variation of coupling from the past (CFTP) technique originally due to Propp and Wilson (1996), we show that it is possible to perform perfect sampling – that is, to sample exactly from the stationary distribution of this class of models – for any specified exit threshold. In particular, we develop an efficient algorithm that generates exact, IID draws from the stationary distribution. For each random seed, the algorithm terminates as soon as an exact draw has been generated, and it is guaranteed to terminate in finite time with probability one. Hence there is no need for the heuristics used to judge burn-in time. Moreover, by repeating the algorithm with independent seeds it becomes possible to generate multiple independent draws from the stationary distribution.

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1 The difficulty of obtaining analytical solutions for this model is related to the existence of a positive threshold for productivity at which firms choose to exit. The threshold introduces a nonlinearity that essentially rules out analytical methods.

2 While some methods for computing error bounds exist, they are rarely used for two reasons. First, they are nontrivial to compute. Second, these bounds are often highly pessimistic, since any such bounds must address the worst case scenario admitted by the model specification.

3 A related issue is that, for a given method, the size of the error is likely to vary with the parameters, since the parameters change the structure of the problem. If the burn-in is not varied accordingly, this is likely to cause bias.

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Our work draws on a large literature on CFTP that mainly exists outside of economics, where the technique is often used for models with large but discrete state spaces.[4] More recently, researchers have developed techniques for implementing CFTP methods in continuous state settings. Murdoch and Green (1998) showed that CFTP can in principle be used in continuous state settings when the underlying Markov process satisfies Doeblin's condition. This condition requires the existence of a nonnegative lower bound function that (a) integrates to a positive value, (b) depends only on the next state, and (c) is pointwise dominated by the transition density function (which depends on both the current state and the next). Theoretical work along the same lines can be found in Foss and Tweedie (1998) and Athreya and Stenflo (2003).

Although these results are fundamental, they can be difficult to apply. Murdoch and Green (1998) admit that their basic method, which is in principle applicable to our model, may have "a limited range of application for two reasons". First, the function associated with Doeblin's condition "may be too small for practical use" to generate exact draws in a reasonable length of time. Second, their method requires the user to draw from scalar multiples of the lower bound transition density and a residual kernel. It can be nontrivial or even impossible to explicitly calculate and draw from these distributions. If approximations are required, this to some degree defeats the purpose of CFTP.

For these reasons, CFTP methods tend to be popular only in specific settings, perhaps the most notable of which is where the underlying Markov process is stochastically monotone. For such processes, efficient and straightforward CFTP methods are available. Corcoran and Tweedie (2001) developed general results on CFTP particularly suitable for monotone Markov processes. An application to economics can be found in Nishimura and Stachurski (2010), where monotonicity makes the algorithm straightforward to implement and analyze.

Here we develop a CFTP algorithm that is designed to produce exact draws from the stationary distribution of Hopenhayn's entry–exit model, which is not monotone. We do however exploit some monotonicity properties from Hopenhayn's model in our algorithm. We show that the algorithm terminates successfully in finite time with probability one by using both the monotonicity of productivity for incumbents and the regenerative property introduced by new entrants. Our algorithm is distinct from Murdoch and Green's method discussed above (Murdoch and Green, 1998), in that it does not use Doeblin's condition, and does not require explicit knowledge of the transition density.[5] As long as one can simulate the overall Markov process, one can sample exactly from the stationary distribution using the algorithm.[6]

Aside from related models of industry dynamics, our techniques can also potentially be applied to other non-monotone regenerative models, such as those found in various intertemporal decision problems. One example is the problem of optimal replacement of a part or machine, the performance of which degrades stochastically over time (see, e.g., Rust, 1987).

2. Preliminaries

2.1. The entry–exit model

In this section we briefly review a benchmark model of firm dynamics due to Hopenhayn (1992). The model is set in a competitive industry where entry and exit is endogenously determined. In the model there is a large number of firms that produce a homogeneous good. The firms face idiosyncratic productivity shocks that follow a Markov process on $S := [0, 1]$. The conditional cumulative distribution function for the shock process is denoted by $F(\phi' \mid \phi)$. Following Hopenhayn (1992), we impose the following restrictions:

Assumption 2.1. F is decreasing in its second argument and, for any $\epsilon > 0$ and any $\phi \in S$, there exists an integer n such that $F^n(\epsilon \mid \phi) > 0$.[7]

We let P denote the stochastic kernel on $[0, 1]$ corresponding to F. That is, $P(\phi, A) := \int \mathbb{1}_A(\phi')F(d\phi' \mid \phi)$ for $\phi \in S$ and $A \in \mathscr{B}$, where $\mathscr{B}$ represents the Borel sets on $[0, 1]$ and $\mathbb{1}_A$ is the indicator function of A. Incumbent firms exit the industry whenever their current productivity falls below a reservation value x_t. Letting M_t be the mass of entrants at time t and ν be the Borel probability measure from which the productivity of entrants is drawn, the sequence of firm distributions $\{\mu_t\}$ on S satisfies $\mu_{t+1}(A) = \int P(\phi, A)\mathbb{1}\{\phi \geq x_t\}\mu_t(d\phi) + M_{t+1}\nu(A)$ for all $A \in \mathscr{B}$. At the stationary equilibrium, both x and M are constant, and a stationary distribution μ is a Borel probability[8] measure μ satisfying

$$\mu(A) = \int P(\phi, A)\mathbb{1}\{\phi \geq x\}\mu(d\phi) + M\nu(A) \quad (A \in \mathscr{B}). \tag{1}$$

It follows from (1) and $\mu(S) = P(\phi, S) = \nu(S) = 1$ that $M = M(x, \mu) := \mu\{\phi \in S : \phi < x\}$. As a result, we can also write (1) as

$$\mu(A) = \int Q(\phi, A)\mu(d\phi) \tag{2}$$

where

$$Q(\phi, A) := P(\phi, A)\mathbb{1}\{\phi \geq x\} + \nu(A)\mathbb{1}\{\phi < x\}. \tag{3}$$

Eq. (2) states that μ is a stationary distribution for the stochastic kernel Q in the usual sense of time invariance. As shown by Hopenhayn (1992), the kernel Q has only one stationary distribution. For the purposes of this paper we will treat x as given. For typical parameter values the stationary distribution has no analytical solution.

2.2. Simulation

It is not difficult to produce an ergodic Markov process suitable for simulation such that its stationary distribution (i.e., time-invariant distribution) coincides with the cross-sectional distribution μ in (2). In essence, we need a method for sampling from the stochastic kernel Q. The first step is to simulate from the conditional distribution $P(\phi, \cdot) = F(\cdot \mid \phi)$. In particular, we seek a random variable U and a function g such that $\mathscr{D}(g(\phi, U)) = F(\cdot \mid \phi)$ for all $\phi \in S$. (Here $\mathscr{D}(X)$ indicates the distribution of random variable X.) This can be achieved via the inverse transform method, where U is uniform on $[0, 1]$ and $g(\phi, u) = F^{-1}(u \mid \phi)$.[9] Now consider the process $\{\Phi_t\}$ defined by

$$\Phi_{t+1} = g(\Phi_t, U_{t+1})\mathbb{1}\{\Phi_t \geq x\} + Z_{t+1}\mathbb{1}\{\Phi_t < x\} \tag{4}$$

where $\{(U_t, Z_t)\}$ is IID with $\mathscr{D}(Z_t) = \nu$ and $\mathscr{D}(U_t) = \text{Uniform}[0, 1]$. In what follows we call (4) the *simulation model*.

Lemma 2.1. *The simulation model is a Markov process with stochastic kernel Q.*

4 Applications range from statistical mechanics to page ranking and the design of peer-to-peer file sharing systems. See, for example, Propp and Wilson (1996), Kijima and Matsui (2006), Huber (2003) and Levin et al. (2009).

5 The assumptions used to show the probability one termination of the algorithm in fact imply Doeblin's condition for some n-step transition, but our proof of this property does not use the latter.

6 Of course in computer implementations exactness is modulo the errors associated with floating point arithmetic and imperfect random number generators.

7 $F^n(\cdot \mid \phi)$ is the conditional distribution for productivity after n periods, given current productivity ϕ.

8 We focus only on normalized measures, since other cases are just scalar multiples.

9 Here $F^{-1}(\cdot \mid \phi)$ is the generalized inverse of $F(\cdot|\phi)$. That is, $F^{-1}(u \mid \phi) := \inf\{z : F(z \mid \phi) \geq u\}$.

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