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Endogenous growth with addictive habits[☆]

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ABSTRACT

In this paper, we investigate the global dynamics of an endogenous growth model with linear technology and addictive habits. We find feasible parameters' conditions under which: (a) the resulting equilibrium consumption path is steeper than in a standard AK model; (b) endogenous fluctuations in the form of damping fluctuations around the balanced growth path emerge; (c) the Easterlin's paradox emerges. The relevance of these results is explained comparing our findings with the results already known in the existing literature.

1. Introduction

In this paper we fully characterize the global dynamics of an AK growth model with addictive external habits. Addiction means that consumption has to be higher than habits, an assumption often used in the literature by considering a subtractive nonseparable utility function (see Boldrin et al. (2001), Constantinides (1990) among others). In this contribution, habits formation is described by the following weighted average of past consumption, with the weights decreasing exponentially into the past:

$$h(t) = \varepsilon \int_{t-\tau}^{t} \bar{c}(u) e^{\eta(u-t)} du$$

where $\varepsilon > 0$, $\eta \geq 0$, and $\tau > 0$ indicate respectively the *intensity*, *persistence* and *lag structure* of the habits, $h(t)$, while $\bar{c}(u)$ is average consumption. This description of habits formation is general relative to the assumptions on their intensity, persistence, and lag structure and it embeds all the main specifications used in the literature (see Remark 1). In particular, there is no contribution in the literature, as far as we know, studying the implication of habit formation in an endogenous growth framework *without imposing additional restrictions* on the parameters ε, η, and τ.[1]

The global dynamics of this model is rich and worth to be investigated for the following reasons. First of all, the presence of consumption aspirations (i.e. $\varepsilon > \eta$, see also later Definition 1) may imply positive growth under the same conditions on the exogenous parameters which would have implied negative growth in the standard AK model (see Proposition 1).[2]

In addition, the global dynamics of an economy, where the positive growth rate is pinned down by consumption aspirations, has the following features: firstly, the consumption equilibrium path is steeper than in a model without consumption aspirations, because such growth rate can be achieved, everything else equal, only if the agents save more at the beginning (see Proposition 1 and Corollary 2); secondly, the growth rate of output and the growth rate of the instantaneous utility may be different and may even have opposite sign (see Proposition 4 and Section 5.3). Therefore, it is possible to specify the exogenous parameters to have constant utility over time but positive output growth—i.e. the model can

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[1] As it will result clear in the following such restrictions rule out two interesting cases: the possibility of a consumption equilibrium path steeper than its counterpart in a model without habit, and the possibility of endogenous damping fluctuations. Also the Easterlin's paradox cannot be obtained with such restrictions.

[2] Of course, the growth rate of the economy has to be lower, in any case, than the real interest rate, otherwise it is not sustainable. A growth rate lower than the interest rate may emerge under some parameters' restriction because we assume constant return to scale in production which implies a constant and positive interest rate.

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easily replicate the so called Easterlin's paradox (see Blanchflower and Oswald (2000) among others). However, it is not possible to use the model to explain this fact and, at the same time, other empirical evidences, such as the equity premium puzzle, where the key parameters in the habits formation equation has to be specified to make the consumption equilibrium path flatter than in the case without habits (i.e. $\varepsilon \leq \eta$ and then no consumption aspiration—see Constantinides (1990) among others).

A full understanding of the global dynamics of this model represents also a contribution to the existing literature on endogenous fluctuations in habits' formation models. In fact, we prove that damping fluctuations around the balanced growth path may arise if the following conditions are met: (i) the habits take the form of consumption aspirations; (ii) the habits are formed over a finite and sufficiently large interval of time (i.e. τ sufficiently large but finite); (iii) the conditions on the parameters would have implied negative growth in a standard AK model (see Propositions 4 and 5 and Section 5.1). Keeping aside the distinction between internal and external habits, our finding differs from the existing contributions because the raising of endogenous fluctuations is proved in an endogenous growth setting, while previous contributions, such as Ryder and Heal (1973) and Benhabib (1978), focused on models without growth, and because of the different source of the fluctuations. In fact, aspiration, a finite lag structure, and a negative modified growth rate of the shadow price of capital are, in our paper, the three necessary ingredients for the endogenous fluctuations around the balanced growth path to emerge while, in the two just cited contributions, endogenous fluctuations depend on the value of the preference discount factor. The source of the endogenous fluctuations in our paper is also different from the one described in Dockner and Feichtinger (1993) because they proved, in a model with a single consumption good which accumulates two stocks of consumption capital, one generating addictive internal habit and the other one satiating behavior, that the "conflicting role" played by the addictive and satiating behavior over the same good is responsible of the raising of endogenous fluctuations.[3]

Finally our analysis represents also a contribution to the large literature on applications of delay differential equations to economic problems, a literature which includes, for examples, vintage capital models (see Boucekkine et al. (1997, 1999, 2005, 1998, 2002) among others), and time-to-build models (see for example, Asea and Zak (1999), Bambi et al. (2012)). In fact, differently from what generally found in these models, the presence of a delay differential equation is not enough by itself to induce endogenous fluctuations since in our model monotonic convergence to the balanced growth path can be found also when the habits formation equation is a delay differential equation.

2. Description of the economy

Consider a standard neoclassical growth model, where the economy consists of a continuum of identical infinitely lived atomistic households, and firms. The households' objective is to maximize over time the discounted instantaneous utility, $u(c(t), h(t))$, which is a function of current consumption, $c(t)$, and external habits $h(t)$. It is indeed assumed, as in Constantinides (1990), and in many others contributions in the macro-finance literature (e.g. Chapman (1998), or one of the specifications in Detemple and Zapatero (1991)), that the instantaneous utility function has the no-separable subtractive form:

$$u(c(t), h(t)) = \frac{(c(t) - h(t))^{1-\gamma}}{1-\gamma}, \tag{1}$$

for $c(t) \geq h(t)$ and $\gamma > 0$ but different from 1. Observe that if $c(t) < h(t)$ then the utility function is not well defined in the real field for some values of γ (e.g. $\gamma = \frac{3}{2}$), and, it is never concave. For this reason, it is generally assumed that $u(c(t), h(t)) = -\infty$ as soon as $c(t) < h(t)$. The instantaneous utility function (1) implies *addictive habits* because current consumption is forced to remain higher than the external habits over time; alternatively it can be seen as a Stone–Geary utility function with an endogenous and time varying subsistence level of consumption, $h(t)$. Our analysis focuses on an exponentially smoothed index of the economy past average consumption rate as a mechanism of habit formation:

$$\begin{aligned} h(t) &= \varepsilon \int_{t-\tau}^{t} \bar{c}(u) e^{\eta(u-t)} du \quad \text{or} \\ \dot{h}(t) &= \varepsilon \left(\bar{c}(t) - \bar{c}(t-\tau) e^{-\eta\tau} \right) - \eta h(t) \end{aligned} \tag{2}$$

where $\eta \geq 0$ measures the persistence of habits, $\varepsilon > 0$ the intensity of habits, i.e. the importance of the economy average consumption relative to current consumption, and finally $\tau > 0$ is the lag structure or memory parameter.[4]

Remark 1. The habit formation equation (2) includes all the specifications of (addictive) habits generally used by the literature; for example:

(a) If $\varepsilon \leq \eta$ and $\tau \to +\infty$, Eq. (2) describes the habits as in Constantinides (1990)[5];
(b) If $\varepsilon > \eta = 0$ and $\tau = 1$, Eq. (2) represents the (continuous-counterpart of the) habits as intended by Boldrin et al. (2001).

It is also worth noting that a choice of τ greater than one and lower or equal than three is consistent with Crawford (2010) econometric estimates. The following definition will turn out useful in the remaining of the paper.

Definition 1 (*Consumption Aspirations*). Habits evolving as in Eq. (2) under the parameters' restriction $\varepsilon > \eta$ are called consumption aspirations.

The representative household solves the following problem where the habits enter as an externality in the instantaneous utility function:

$$\begin{aligned} &\max \int_0^\infty \frac{(c(t) - h(t))^{1-\gamma}}{1-\gamma} e^{-\rho t} dt \\ &s.t.\ \dot{k}(t) = (R(t) - \delta) k(t) - c(t) \\ &\quad k(t) \geq 0,\ c(t) \geq h(t) \geq 0 \\ &\quad k(0) = k_0 > 0 \end{aligned}$$

with $R(t)$ the rental rate of capital, and δ the depreciation rate. From now on we will assume a priori that the control is interior and the state constraint is satisfied. Then we will identify a posteriori what are the restrictions to be imposed so that all the inequalities constraints hold. Following this strategy, we may easily write the first order conditions from the present value Hamiltonian

$$c(t) - h(t) = \varphi(t) \tag{3}$$

$$\frac{\dot{\varphi}(t)}{\varphi(t)} = \frac{1}{\gamma}(r(t) - \rho) \tag{4}$$

[3] The example provided by these authors is indeed very illustrative: "only a consumer with a desire to eat and a dislike for weight who anticipates the future consequences of his current actions can end up in eating and dieting cycles".

[4] The external habits at two sufficiently far dates may be completely unrelated under a finite choice of τ; in fact, only the average consumption between t and $t-\tau$ matters: in this respect, a finite τ introduces a complete and periodic update of the habit.

[5] Gómez (2014) has recently investigated the global dynamics of a model with external habits but without habit addiction. Also the specification of the habit formation is different since it is assumed (using our notation) that $\varepsilon = \eta$, and $\tau = \infty$. Other relevant contributions which have studied the dynamics of habits formation models without addiction are Carroll et al. (1997a,b).