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Ignorance and competence in choices under uncertainty☆

Paulo Casaca[a,b], Alain Chateauneuf[c,d], José Heleno Faro[e,*]

[a] FIEMG, Av. do Contorno 4456, Funcionários 30110-916, Belo Horizonte, Brazil
[b] Cedeplar/FACE-UFMG, Av Antônio Carlos 6627, 31270-901, Belo Horizonte, Brazil
[c] IPAG Business School, 184 bd Saint-Germain 75006 Paris Cedex 05, France
[d] Paris School of Economics, U. de Paris I, 106-112 bd de l´Hopital, 75647 Paris Cedex 13, France
[e] Insper Institute of Education and Research, Rua Quatá 300, Vila Olímpia 04546-042, São Paulo, Brazil

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ABSTRACT

We propose a model of decision making that captures reluctance to bet when the decision maker (DM) perceives that she lacks adequate information or expertise about the underlying contingencies. On the other hand, the same DM can prefer to bet in situations where she feels specially knowledgeable or competent even if the underlying contingencies have vague likelihoods. This separation in terms of sources of uncertainty is motivated by the Heath and Tversky's competence hypothesis as well as by the Fox and Tversky's comparative ignorance effect. Formally, we characterize preference relations $\succsim$ over Anscombe–Aumann acts represented by

$$J(f) = \min_{p\in C} \int_A u(f)dp + \max_{p\in C} \int_{A^c} u(f)dp,$$

where u is an affine utility index on consequences, C is a nonempty, convex and (weak*) compact subset of probabilities measures, and A is a *referential chance event*. In this model there is a clear separation of ambiguity attitudes. The case $E \subset A$ captures possible familiar target events while the case $E \subset A^c$ might refer to the case of relative ignorance concerning related contingencies. This model captures a special case of event dependence of ambiguity attitudes in which the well known maxmin model is a special case. We also characterize the case where we have a Choquet Expected Utility representation. *Journal of Economic Literature Classification Number*: D81.

1. Introduction

Motivated by the well-known Ellsberg paradox (1961), ambiguity became an important issue in decision theory modeling sensibility to the lack of precise probabilistic information. The most well known models capturing ambiguity sensitivity are given by preference relations with a non-additive functional representation, as in Schmeidler (1989)'s Choquet Expected Utility (CEU) and Gilboa and Schmeidler (1989)'s Maxmin Expected Utility (MEU) models. In this perspective, the classical additive case of Subjective Expected Utility (SEU) of Savage (1954) (or Anscombe and Aumann, 1963) imposes strong behavioral conditions on preferences, which includes independence, that implies an insensitive or neutral attitude towards ambiguity.[1]

The widely discussed hypothesis that *people prefer to bet on known rather unknown probabilities* is the basis for the notion of ambiguity aversion (uncertainty aversion). For instance, this hypothesis is essential in the MEU model where a DM behaves *as if* he had a set of probability measures that determinates his *ex ante*

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* Corresponding author. Tel.: +55 11 45 04 24 22.
E-mail addresses: josehf@insper.edu.br, jhfaro@gmail.com (J.H. Faro).

[1] Ghirardato and Marinacci (2002) provided a complete characterization of a comparative notion in which the SEU model is the benchmark of ambiguity neutrality.

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valuation of any act by the corresponding worst expected utility.[2] Although ambiguity aversion presents many interesting applications in economic problems,[3] the generality of this pattern of attitude towards ambiguity is questionable.[4] Heath and Tversky (1991) discussed another pattern of behavior where a DM might prefer to bet in a context that she considers herself competent than in a context where she feels ignorant or uninformed. Here, the term competence is used in a broad sense that includes skill, knowledge or understanding. This ideas motivate Heath and Tversky (1991) to propose the "competence hypothesis" asserting that the DM's willingness to bet on an event depends not only on the estimated likelihood and the precision of that estimate, but also on her general knowledge or understanding of the relevant context. In the widely discussed Ellsberg urns, we have the situation of partial ignorance characterized the inability of improving the knowledge of the proportion of balls in the urn. Fox and Tversky (1995) extended the Heath and Tversky's analysis by asking what conditions produce ignorance aversion. The main idea in Fox and Tversky (1995) is that the DM's confidence betting on a target event is enhanced (diminished) when she contrasts her knowledge of the event with her inferior (superior) knowledge about another event, or when she compares himself with less (more) knowledgeable individuals. In this way, the "comparative ignorance hypothesis" of Fox and Tversky (1995) asserts that the ambiguity aversion is driven by a comparison with more familiar sources of uncertainty or expert and it is diminished in the absence of such a comparison. Also, following again Heath and Tversky (1991), in many situations the DM's perception of her level of knowledge concerning a target event might be extremely positive and that case she also may prefer to bet on her vague assessment of familiar events rather than bet on chance events with matched probability.

We aim to focus on the cases where an event A is a *clear and unambiguous reference* for the DM in terms of familiar or unfamiliar contingencies. We interpret the event A as the possible domain of unfamiliarity or ignorance, while its complement A^c corresponds to possible familiar or expert knowledge. The main idea is that under unmeasurable uncertainty about sub-events of A, the DM features ambiguity aversion with respect to contingent payoffs in A. On other hand, when uncertainty over sub-events of A^c is not objectively measurable, the DM features an ambiguity preferring attitude about contingent payoffs in A^c. Following again Heath and Tversky (1991), we might also interpret such separation across the event A and A^c by attributes for success and failure as depending on the uncertainty context. In the domain of chance or risk determined by the assumed well specified probability of A, where an act f pays a consequence x on A and a consequence y on A^c, both success and failure are attributed primarily to luck in a classical sense of roulette wheels. On the other hand, if the DM has limited understanding about the problem at hand in terms of likelihood when comparing payoffs over A, failure can be attributed to the ignorance or the lack of expertise and might prevent the DM from taking credit for success (relative good states), but exposes him to blame in the case of failure (relative bad states). In contrast, if a DM is an expert and behaves as ambiguity seeking with respect to payoffs in A^c, success is attributable to the knowledge, whereas failure can be attributed to chance, which helps him to take credit when they succeed and provides protection against blame when he fails. Next, we illustrate situations in which the DM has a clear referential chance event that separates his possible patterns of behavior due to different sources of uncertainty:

Example 1. It concerns quarterfinals in a World Cup composed of four South American teams and four European teams. A commentator is an expert on South American soccer and has to bet on the next Champion of the Word Cup. Also, the commentator feels unfamiliar within European teams. Using the retrospect of the previous World Champions, the commentator considers that each continent has the same chance to win the World Cup. Moreover, assume that the commentator is not able to assign a well defined probability for each team to be the champion. In this case, the results are ambiguous and they allow the commentator to behave differently depending on the event considered. Here, we can consider the state space $S = \{a_1, \ldots, a_4, e_1, \ldots, e_4\}$, where each a_i denotes a South American team and each e_i denotes a European team, and the domain of familiarity or expertise is given by $A^c = \{a_1, \ldots, a_4\}$.

Example 2. A pulmonologist receives a patient with an undiagnosed disease. Before any exam, he will give some hypothesis in order to guide subsequent procedures. Despite his expertise in respiratory problems, the disease might be also cardiac. In the preliminary diagnosis, the doctor needs to take into account whether the disease could be respiratory or not. Without accurate exams the precise disease cannot be determined on probabilistic terms. Analyzing the frequency of patients with problems related to other specialties in his office, the pulmonologist considers a probability of 70% for a disease associated with the respiratory system. If the disease pointed out by the doctor is related to the respiratory system, he will be *optimistic about his prognostic and appropriateness of the treatment indicated*. However, if the prognostic is related to the cardiac system, then he will be *pessimistic about his judgment*. In this case, he can suggest a specialist in cardiological diseases for providing another opinion. In this case, A^c is the set of states where the patient is found to have a respiratory disease.

Example 3. A stock broker specialist in technology firms is hired by a stock brokerage. We assume that the stock broker feels familiar in only handling stocks related to technology firms. This tech-stock investor should bet on the best performer among a set of ten stocks, where some are technology and some are not (assume, for simplicity, only commodity stocks). Assume that 30% of the time the better performance is related to the technologic firm. Nevertheless, it is unclear which is the likelihood associate to all specific assets to be the best. Here, A^c is the set of states in which a tech stock is the best performer.

We note that the common feature in the previous examples is the existence of some referential event, which can viewed as an objective information that DMs have obtained before making the decision. The intuition is that DMs modeled here can claim that with a certain probability the decision problem leads to some contingence related to their expertise while the complementary probability contemplates unfamiliar states. Note also that we are assuming that decision making takes place in a *small world* (Savage, 1954, p. 9) in the sense that events are relevant to a particular decision situation. Hence, in our motivation we vary the set A with the application.[5,6]

[2] Indeed, Cerreia-Vioglio et al. (2011a) provided a representation result for uncertainty averse preferences under a very weak notion of independence called "risk independence". For instance, special cases are given by Chateauneuf and Faro (2009) and Maccheroni et al. (2006). See also Cerreia-Vioglio et al. (2011b).

[3] See, for instance, Section 6 in Gilboa and Marinacci (2011).

[4] An interesting discussion on this topic is presented in Fox and See (2003), where many interesting references are also provided.

[5] In the case where we want to allow a notion of *universal setS* it would be possible to consider $S = S_1 \times \cdots \times S_n$, where each S_i, $i = 1, \ldots, n$, takes the form $S_i = A_i \cup A_i^c$ and we have preferences $\succsim_i$ over acts $f : S_i \to X$ as characterized in this paper. How to extend all preferences $\succsim_i$ to a common one over all acts defined on S is a natural question. One possible way is to declare that the DM has incomplete preferences and applies a kind of Paretian criteria or unanimity rule using all $\succsim_i$ over each S_i for raking them.

[6] An interesting approach dealing with a model of small words was proposed by Chew and Sagi (2006, 2008) by providing an axiomatic model of source preferences. In this model different attitudes towards risks can arise from distinct sources of uncertainty.

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