Journal of Mathematical Economics 54 (2014) 151–156

Contents lists available at ScienceDirect

Journal of Mathematical Economics

journal homepage: www.elsevier.com/locate/jmateco

Short communication

Backward induction and unacceptable offers[☆]

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ARTICLE INFO

Article history:
Received 15 March 2013
Received in revised form
16 October 2013
Accepted 27 January 2014
Available online 7 February 2014

Keywords:
Bargaining
Negotiation
Irrevocable breakdown
Delay

ABSTRACT

How to establish the existence of subgame perfect equilibrium (SPE) in bargaining models if no stationary SPEs (SSPEs) exist? The backward-induction technique of Shaked and Sutton (1984, Econometrica) applies to the cyclical structure of SPE payoffs and provides recursive dynamics on the bounds of SPE payoffs. Acceptable and unacceptable offers have to be incorporated for these dynamics to be necessary and sufficient for extreme SPEs. In this paper, we demonstrate how these recursive dynamics are directly applicable to establish the existence of SPE in a model with no SSPE. Also from these dynamics, the extreme SPE strategy profiles can easily be recovered.

1. Introduction

Stationary subgame perfect equilibrium (SSPE) receives a lot of attention in the bargaining literature for variety of reasons.[1] For example, an SSPE has a simple structure; it is often unique and relatively easy to compute. The seminal bargaining models of Rubinstein (1982) and Shaked and Sutton (1984) have a unique SPE that is also stationary. Stationary strategy profiles are also commonly adopted in establishing the existence of SPE. When an SSPE does not exist, such as in Vidal-Puga (2008) and Herings and Houba (2010), there seems to be a lack of a systematic method to analyze SPEs, in particular to establish the existence of SPE. In this paper, we advocate a modification of the backward-induction technique of Shaked and Sutton (1984) as this systematic method. The reason is that, even if no SSPE exists and all SPE strategy profiles are necessarily nonstationary, the set of SPE payoffs still is cyclical, in particular the extreme SPE payoffs are cyclical.[2] The backward induction technique of Shaked and Sutton (1984) exploits the cyclical structure of the extreme SPE payoffs and, thereby, circumvents having to perform the equilibrium analysis in the space of nonstationary strategy profiles. If one succeeds in characterizing the extreme SPE payoffs, then one has also established the existence of SPE without resorting to stationary strategy profiles. In turn, the extreme SPE payoffs characterize the entire set of SPE payoffs and, from the backward-induction dynamics, one can easily recover all the (possibly nonstationary) extreme SPE strategy profiles. Although the latter two facts are well known, there is no literature that considers the backward induction technique of Shaked and Sutton (1984) as a systematic road map to establish the existence of an SPE, in particular if no SSPEs exist.

Given the existence of an SSPE in Rubinstein (1982), Shaked and Sutton (1984) provide an alternative and intuitive proof for the uniqueness of SPE by backward induction on the bounds of the SPE payoffs. Uniqueness in SPE payoffs follows if each player's lowest SPE payoff is equal to his highest SPE payoff. Shaked and Sutton (1984) focus on the case where the proposing player always offers at least the continuation payoff to the responding player and the responding player always accepts such offers. In other words, they exclude that a proposing player may make unacceptable offers—offers to the responding player that are less than the responding player's lowest possible continuation payoff and, hence, that will surely be rejected. By excluding unacceptable offers, Shaked and Sutton (1984) only provide necessary conditions for the extreme SPE payoffs and, for the model in Rubinstein (1982), there is no loss of generality because these conditions already imply uniqueness in SPE payoffs. Fudenberg and Tirole (1991) and Muthoo (1999) formally incorporate unacceptable offers in the original argument of

☆ We would like to thank Thijs den Hollander, the editor in charge and the referee for their comments and suggestions.

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1 Equilibria in stationary strategy profiles require that each player acts the same whenever this player faces the same continuation game; see e.g. Harsanyi and Selten (1988) or Maskin and Tirole (2001). For an axiomatic foundation of stationary equilibria, we refer to Bhaskar et al. (2013).

2 By extreme SPE payoffs, we mean a player's highest and lowest SPE payoffs.

http://dx.doi.org/10.1016/j.jmateco.2014.01.013
0304-4068/

Shaked and Sutton (1984) and, therefore, they provide the necessary and sufficient conditions for the extreme SPE payoffs in Rubinstein (1982). Although these necessary and sufficient conditions are applicable, the possibility to establish the existence of SPE in Rubinstein (1982) directly from these conditions was simply never realized due to the well-known SSPE in his model. In addition, there is no need for these necessary and sufficient conditions in this particular model because the necessary conditions of Shaked and Sutton (1984) alone are already enough to establish the uniqueness of SPE.

Since the early literature, backward induction has become a standard tool for deriving the extreme SPE payoffs in bargaining models. It is not only used to establish the uniqueness of SPE in e.g. Furusawa and Wen (2002), but also to derive the extreme SPEs directly in e.g. Haller and Holden (1990), Fernandez and Glazer (1991), Busch and Wen (1995), Merlo and Wilson (1995) and Houba and Wen (2011). The last two references demonstrate the necessity of including both acceptable and unacceptable offers in the analysis of extreme SPE payoffs. These references once more demonstrate the generality of modifying the original argument of Shaked and Sutton (1984); the idea to establish the existence and characterize all SPEs directly upon these necessary and sufficient conditions is novel. Without including unacceptable offers, the conditions obtained are only necessary; they may not be sufficient to establish the existence of SPE or mislead in characterizing the extreme SPEs.

The main objective of this paper is to advocate the backward-induction technique of Shaked and Sutton (1984), of course after modifying it for unacceptable offers, as an effective tool to systematically establish the existence of SPE, especially when an SSPE does not exist. The bargaining model of Vidal-Puga (2008) is a perfect example to demonstrate this due to the following facts: first, this model does not admit an SSPE for some parameter values. Second, depending upon the parameter values, there are SPEs where unacceptable offers are crucial to characterize the extreme SPEs. Third, Vidal-Puga (2008) obtains closed-form solutions for the extreme SPE payoffs by considering separate cases, while we obtain the same solutions directly from the unified backward-induction dynamics. We then demonstrate how to recover the extreme SPE strategy profiles from these backward-induction dynamics, especially when no SSPE exists. Finally, our analysis does provide a novel explanation why unacceptable offers arise in Vidal-Puga (2008) in supporting the extreme SPEs for some parameter values: although delay is inefficient, there is no mutually beneficial agreement where the proposing player receives at least his continuation value from any unacceptable offer and the responding player receives at least his payoff from any acceptable offer.

The rest of this paper is organized as follows. Section 2 presents the bargaining model of Vidal-Puga (2008). In Section 3, we derive the backward-induction dynamics for the extreme SPE payoffs by including the possibilities of making acceptable as well as unacceptable offers. In Section 4, we demonstrate how easy it is to recover the strategy profiles that support these extreme SPEs from the backward-induction dynamics.

2. The model

Two players, A and B, bargain for an agreement to share a pie of size 1. There are infinitely many periods and the two players alternate in making offers and counter offers until they reach an agreement. Denote the first period as period 0, in which player A makes an offer. During each period before an agreement is reached, one player, the proposer, makes an offer (x_A, x_B), where $x_A + x_B \leq 1$. The other player, the responder, decides whether to accept or reject the offer. If the responder accepts the offer, then both players receive their shares of the pie in the offer, respectively, and the game ends. If the responder rejects the offer, then an exogenous random draw will determine whether the standing offer (x_A, x_B) is final. With probability $\rho \in [0, 1]$, bargaining proceeds to the next period where the current responder becomes the proposer and makes a counter offer. With probability $1 - \rho$, the standing offer (x_A, x_B) becomes final and the current responder has a second chance to accept or reject the offer. Both players receive their respective shares in the offer if the responder accepts the final offer, or nothing otherwise, and the game ends. Every player discounts his payoff by discount factor $\delta \in [0, 1)$ per period; player i's payoff from an agreement (x_A, x_B) reached in period t is $\delta^t x_i$ for $i = A$ and B.

The bargaining model described above is a well-defined extensive-form game with perfect information. As two special and extreme cases, it is equivalent to the alternating-offer bargaining model of Rubinstein (1982) when $\rho = 1$, and it is the ultimatum game when $\rho = 0$. It is well known that there is a unique subgame perfect equilibrium (SPE) with an immediate agreement in either of the two extreme cases. However, the equilibrium agreements in the two extreme cases are very different: Player A, the proposer in the first period, receives $\frac{1}{1+\delta}$ in the alternating-offer bargaining model of Rubinstein (1982) for $\rho = 1$, and 1 in the ultimatum game for $\rho = 0$. Vidal-Puga (2008) shows how the subgame perfect equilibrium agreement varies with respect to the discount factor δ and the probability of continuation ρ, and the existence of multiple equilibrium payoffs for some intermediate values of δ and ρ.

3. Backward induction

As discussed in the introduction, our objective is to demonstrate the effectiveness of backward induction in characterizing the bounds of equilibrium payoffs in bargaining models, in particular when making unacceptable offers is necessary to support extreme equilibria, such as Shaked and Sutton (1984). We will show not only how to include unacceptable offers directly into the backward induction technique, but how to recover systematically the extreme equilibrium strategy profiles from the backward induction. Generally speaking, in order to obtain the necessary and sufficient conditions for the bounds of the equilibrium payoffs to the proposer and responder, total eight payoff bounds, we need to analyze players' strategy profiles over two consecutive periods. In bargaining problems where the proposer/responder faces the same continuation game, the analysis can be greatly simplified by using this type of characteristic in many bargaining models.

Assume that the set of equilibrium payoffs in the bargaining model described in the previous section is non-empty and compact.[3] Observe that the proposer, either player A or player B, faces the same continuation game, and the responder, either player A or player B, also faces the same continuation game when responding to the same offer. Therefore, whether a player is the proposer or responder is vital to the analysis, but the identities of the proposer and responder are irrelevant. Let M_P and m_P be the maximum and the minimum of the proposer's SPE payoffs at the beginning of a period. Let M_R and m_R be the maximum and the minimum of the responder's SPE payoffs in the beginning of a period. By definition,

$$m_P \leq M_P \quad \text{and} \quad m_R \leq M_R.$$

We refer to any SPE in which a player receives his maximum/minimum equilibrium payoff as this player's best/worst SPE. Without loss of generality, we focus on a period where player A makes an offer to player B. Note that if player A offers (x_A, x_B), player B will definitely

- accept offer (x_A, x_B) if $x_B > (1-\rho)x_B + \rho\delta M_P \Leftrightarrow x_B > \delta M_P$,
- reject offer (x_A, x_B) if $x_B < (1-\rho)x_B + \rho\delta m_P \Leftrightarrow x_B < \delta m_P$.

[3] Applying the technique of self-generating equilibrium payoffs by Shaked and Sutton (1984) and Binmore (1987), one can show that the set of SPE payoffs is compact.