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Learning, pricing, timing and hedging of the option to invest for perpetual cash flows with idiosyncratic risk

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ABSTRACT

The paper considers the option of an investor to invest in a project that generates perpetual cash flows, of which the drift parameter is unobservable. The investor invests in a liquid financial market to partially hedge cash flow risk and estimation risk. We derive two 3-dimensional non-linear free-boundary PDEs satisfied by the utility-based prices of the option and cash flows. We provide an approach to measure the information value. A numerical procedure is developed. We show that investors have not only idiosyncratic-risk-induced but also estimation-risk-induced precautionary saving demands. A growth of estimation risk, risk aversion or project risk delays investment, but it is accelerated if the project is more closely correlated with the market. Partial information results in a considerable loss, which reaches the peak value at the exercising time and increases with project risk and estimation risk. The more risk-averse the investor or the weaker the correlation, the larger the loss.

1. Introduction

The paper considers the option of an investor to invest in a non-tradable irreversible project that generates perpetual cash flows, of which the drift parameter is unobservable (partial information). We assume the investor hedges the estimation risk and cash flow risk by investing in a liquid financial market. However, generally speaking, an investor is still exposed to considerable unhedged idiosyncratic risk and therefore, we price the real option and cash flows by consumption utility indifference pricing approach.

We study the investor's joint decisions of investment for perpetual cash flows, consumption/savings, and portfolio selection when he cannot fully insure the cash flow shocks and needs to learn about the uncertain drift parameter. As a result, risk attitude, idiosyncratic risk, and the subjective estimate of the drift parameter have substantial effects on the decisions.

Applying the consumption utility indifference pricing method, continuous-time stochastic control and filtering theory, we derive a system of high-dimensional non-linear free-boundary PDEs (Partial Differential Equations) for the implied values of the real option and cash flows. We develop an effective finite difference procedure, which allows us to present an extensive analysis with regard to the impact of learning about uncertainty on the pricing, timing and hedging of the option to invest.

Our contributions

Intuitively, unlike the full information case (i.e. the drift is observable), an investor with partial information would very likely make a "wrong" decision and thus incur a loss. It is interesting to measure the loss since it can be considered as the maximum cost an investor would like to pay in order to obtain the full information. Naturally, we call the loss the implied information value. However, there are almost no papers to measure the quantity of the loss in the real options literature including Décamps et al. (2005) among others. In this paper, we provide an approach to quantify the loss. It surprises us that the partial information leads to a considerable decrease (loss) in the implied value of the option to invest, i.e. the implied information value is significant relative to investment cost (sunk cost).

Our analysis indicates that the implied information value reaches the maximum value at investment threshold and it increases with risk aversion, project risk, and prior variance. Their growths also considerably raise the precautionary saving motive, decrease the certainty-equivalent wealth of cash flows, and delay real investment. Investors, particularly ones with partial information, are still exposed to the idiosyncratic risk of cash flows after investment, though the systematic risk can be hedged away by investing in a liquid financial market. Consequently, in contrast

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to Décamps et al. (2005), learning about the uncertain drift parameter is valuable all the time no matter whether the option is exercised or not, and a more effective estimate of the drift remarkably increases the implied values of both the option and cash flows, speeding up investment.

Unlike Miao and Wang (2007) and Décamps et al. (2005), our results provide four new insights into the irreversible investment for perpetual cash flows with an unobservable drift and idiosyncratic risk. First, investors have not only idiosyncratic-risk-induced but also have estimation-risk-induced precautionary saving demands both before and after investment. Second, a growth of estimation risk, risk aversion or project risk delays investment. Third, investment is accelerated if the project is more closely correlated with the market. Last but more importantly, partial information results in a considerable loss, which reaches the peak value around the investment threshold and increases with project risk and estimation risk. The more risk-averse the investor or the weaker the correlation, the larger the loss.

Furthermore, we develop an efficient finite difference procedure to solve the system of three-dimensional non-linear free-boundary PDEs that characterize the model solutions. This is numerically more challenging than the one-dimensional problem considered by Miao and Wang (2007), and the two-dimensional problems discussed by Décamps et al. (2005), Yang and Yang (2012) and Yang et al. (2011) due to the high non-linearity in two spatial dimensions. The additional complexity arises from the dependence of the filtering estimate of the drift not only on cash flows, but also on the market portfolio.

Literature review

Real investment decisions play a fundamental role in entrepreneurial activities and modern economics. A real investment is typically irreversible with uncertain future rewards and flexible investment time. The right to decide when to invest in a project is analogous to an American style financial option and so it is called real option. The real options approach to investment under uncertainty originates from the work of Myers (1977) and presently becomes more popular. Major contributions along this research line are McDonald and Siegel (1986) and Dixit and Pindyck (1994) among others. Recently, Henderson and Hobson (2002), Miao and Wang (2007), Henderson (2007) and Ewald and Yang (2008) study the real options problem under incomplete markets by utility indifference pricing approach. However, almost all papers including Miao and Wang (2007) in the literature assume that an investor has access to full information. Under this assumption, the mean appreciation rate of the value or cash flows of a project and the driving Brownian motion are observable, which is of course unrealistic. Following Yang and Yang (2012) and Song and Yang (2013), the feature of this paper is that in contrast to the above papers, we relax this assumption to suppose that the investor cannot observe the drift parameter and the Brownian motion appearing in the stochastic differential equation describing the cash flows. In other words, we assume an investor has only access to partial information, as argued by Gennotte (1986), Lakner (1998), Brennan (1998), Yang and Ma (2001), Xiong and Zhou (2007), Monoyios (2007), Monoyios (2008), Wang (2009) among others.

The "partial information" assumption in our model is quite realistic since the drift parameter and the paths of Brownian motions are fictitious mathematical tools, which are of course not observable. On the contrary, the volatility/dispersion parameter for the cash flows will be observable since one can prove that the volatility is adapted to the filtration generated by the cash flows.

Our model is closely related with Décamps et al. (2005) and Klein (2009) since the two papers also discuss the real options problems with partial information. But the distinction between them and our paper are also evident: First, we suppose that the drift parameter follows a normal distribution other than a two-point distribution as assumed by them. Second and the most importantly, we solve the real options problem based on consumption utility indifference pricing approach, while they assume that investors are risk-neutral. Taking into account that a real investment is generally exposed to considerable idiosyncratic risk, this difference makes our problem more interesting and naturally more challenging as well. We fill the gap by developing a comprehensive model and a numerical method for an investor who has to deal with both estimation risk and idiosyncratic risk resulting from cash flows.

To the best of our knowledge, this paper is most related with Yang et al. (2011), Yang and Yang (2012) and Song and Yang (2013). However, we assume in this paper that an investor obtains stochastic cash flows rather than a lump-sum payoff upon investment as assumed by Yang and Yang (2012) and Song and Yang (2013). This distinction is trivial in a risk neutral world but significant in our model since we suppose that the investor is risk-averse and thus, one cannot get an equivalent lump-sum payment simply by discounting future cash flows. On the other hand, although Yang et al. (2011) do consider the situation where the project generates cash flows, they make the assumption that the investor has only access to one risk-free asset. In our paper, aside from the risk-free asset, there exists another tradable risky asset (e.g. market portfolio) in a liquid financial market which can partially hedge the cash flow risk and estimation risk. This difference is fundamental because the problem we discuss here is more realistic, interesting and challenging. In this way, we can obtain more insightful conclusions. For example, once the absolute value of the correlation between the market portfolio and the project goes up, the investor will hedge more risk of the cash flows and the remaining idiosyncratic risk becomes less, which decreases his precautionary savings demand, naturally raises the implied value of the option and speeds up the real investment as well.

In fact, our paper undertakes a systematic investigation of learning and hedging and many conclusions here have not been addressed yet. Unlike the previous papers, for instance, the learning in the present paper has implications not only on the implied option value of waiting, but also on the certainty-equivalent wealth of perpetual cash flows after investment. More importantly, the estimation risk from learning and the idiosyncratic risk left after partial hedging have effects on hedging and precautionary saving demands, which affect investment decisions and the implied information value.

Furthermore, cash flow volatility has three effects on the investor's decisions in this study. First, the volatility increases the option value because of the standard asymmetric convex payoff of an option. The second effect is that idiosyncratic volatility instead of project volatility induces the standard precautionary saving demand against cash flow fluctuations. Third, the idiosyncratic volatility rather than project volatility *increases* the estimation risk induced by learning about the drift parameter for a given fixed prior variance. This finding is implied by our filtering results and is also confirmed by the effects of the project volatility and prior variance as is evident from our numerical results. The interpretation is that the realized cash flows with higher idiosyncratic volatility make it more difficult to estimate the drift effectively and the estimation risk is increased accordingly, which naturally induces a larger precautionary saving demand.

Finally, the study incorporating learning and hedging is also particularly challenging. The main additional difficulty is that, the filtering estimate of the drift parameter depends not only on the observations of cash flows, but also on the value of the market portfolio. Consequently, solving a combined stochastic control and optimal stopping problem of investing for cash flows leads to a system of three-dimensional non-linear free-boundary PDEs. This is more complicated than a two-dimensional PDE in Yang and Yang