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Equilibrium dynamics in a class of one-sector endogenous growth models with external habits: An application of special functions[☆]

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HIGHLIGHTS

- We consider a class of one-sector endogenous growth models with external habits.
- We characterize the time paths of the levels of consumption, habits and capital.
- Levels may exhibit non-monotonic dynamics, in spite of monotonicity of their ratios.
- Our results support the use of special functions in the study of economic dynamics.

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ABSTRACT

This paper analyzes the equilibrium dynamics in a class of one-sector endogenous growth models with external habits. Using an explicit solution expressed in terms of the Gauss hypergeometric function, we show that the levels of consumption, habits and capital may exhibit non-monotonic transition dynamics, even though their ratios converge monotonically. A numerical simulation illustrates this result.

1. Introduction

Habit-forming preferences have been widely used in the literature (e.g., Abel, 1990 and Constantinides, 1990). In habit-formation models, individual's utility depends not only on her own consumption but also on how it compares to a reference consumption level—the habits stock. Starting from Carroll et al. (1997), several works have studied the dynamics of the AK model with habits (e.g., Carroll et al., 2000, Mansoorian and Michelis, 2005, Alonso-Carrera et al., 2006, Gómez, 2006 and Chen, 2007). These studies analyzed the dynamics of ratios between the original variables—states, costates and controls. However, as argued by Boucekkine and Ruiz-Tamarit (2008), this 'dimension reduction' strategy has the weaknesses that the dynamics of levels remain largely unknown.

This paper studies the transition dynamics in a class of one-sector endogenous growth models with habits. Following Boucekkine and Ruiz-Tamarit (2008), we use an explicit solution expressed in terms of the Gauss hypergeometric function. Unlike previous works, the global dynamics of levels, rather than just ratios, are characterized analytically. Thus, we can show that consumption, habits and capital may exhibit non-monotonic dynamics, even though their ratios converge monotonically. An example illustrates this result.

The model considered is a special case of the existing literature. First, as in Carroll et al. (1997, 2000), we assume that habits enter utility multiplicatively. Other commonly used specification introduces habits additively (e.g., Constantinides, 1990). Second, we consider external habits that are formed from economy-wide average past consumption. Carroll et al. (1997, 2000) also consider internal habits that depend on individual's own past consumption. Third, following Carroll et al. (1997, 2000), we consider an AK-type technology. This is appealing because it allows isolating the effect of habits. Other authors studied the neoclassical growth model (Alonso-Carrera et al., 2005) or the non-scale growth model (Alvarez-Cuadrado et al., 2004) with habits. However, an analytical study is probably intractable in these models because of the

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complexity of the ensuing dynamic systems, so they had to resort to a local study combined with numerical simulations.

This paper is also related to a still small literature that uses special functions to analyze the dynamics of growth models. In particular, hypergeometric functions have been used to study, e.g., the Solow and the AK models with logistic population (Guerrini, 2006, 2010) and the two-sector Lucas model (Boucekkine and Ruiz-Tamarit, 2008; Hiraguchi, 2009; Boucekkine et al., 2013). This paper, therefore, provides more evidence that special functions are useful in economic dynamics.

The rest of this paper is organized as follows. Section 2 presents the model. Section 3 analyzes the equilibrium dynamics and provides an example. Section 4 concludes.

2. The model

We consider a closed economy inhabited by identical infinitely-lived agents. Population at time t, $N(t)$, grows at a constant rate $n \geq 0$. The agent derives utility from consumption, $C(t)$, and a reference consumption level or habits stock, $H(t)$, according to

$$U = \int_0^\infty \frac{[C(t)H(t)^{-\gamma}]^{1-\epsilon} - 1}{1-\epsilon} e^{-\theta t} dt, \quad \epsilon > 0,\ \theta > 0,\ 0 < \gamma < 1, \tag{1}$$

where θ is the rate of time preference. The (external) habits stock evolves according to

$$\dot{H}(t) = \rho[\bar{C}(t) - H(t)], \qquad H(0) = H_0 > 0, \quad \rho > 0. \tag{2}$$

Here, $\bar{C}(t)$ is the economy-wide average consumption, which is taken as given by the agent, and ρ governs the speed with which habits adjust to consumption.

Output per capita is $Y(t) = BK(t)^\alpha G(t)^{1-\alpha}$, $B > 0$, $0 < \alpha \leq 1$, where $K(t)$ is the capital per capita and $G(t)$ is the public expenditure per capita. The government levies a flat-rate tax on income, τ, and runs a balanced budget, $\tau Y(t) = G(t)$, with $0 < \tau < 1$ if $\alpha < 1$, and $0 \leq \tau < 1$ if $\alpha = 1$. Thus, this model encompasses the Barro model and the standard AK endogenous growth model with habits. Let $\delta \geq 0$ denote the rate of capital depreciation. The agent maximizes (1) subject to her budget constraint

$$\dot{K}(t) = (1-\tau)Y(t) - C(t) - (n+\delta)K(t),$$

taking as given the average consumption, $\bar{C}(t)$, the stock of habits, the income tax and public expenditure, and the initial condition on capital, $K(0) = K_0 > 0$.

An equilibrium is as a set of paths $\{C(t), K(t), H(t)\}_{t=0}^\infty$ that solves the agent's problem, such that the habits stock evolves according to (2), the government obeys its budget constraint, and $\bar{C}(t) = C(t)$ for all t. Using the government budget constraint to solve for $G(t)$, equilibrium output can be determined as $Y(t) = \tau^{(1-\alpha)/\alpha} B^{1/\alpha} K(t)$.

Let us define $c \equiv C/H$, $h \equiv H/K$, $A = \alpha(1-\tau)\tau^{(1-\alpha)/\alpha}B^{1/\alpha} - n - \delta$, and $\bar{A} = (1-\tau)\tau^{(1-\alpha)/\alpha}B^{1/\alpha} - n - \delta$. The following result describes the dynamics of the economy.[1]

Theorem 1. *The economy has a unique interior equilibrium with positive long-run growth which is described by*

$$C(t) = (c_0/\hat{c})^{\frac{\epsilon}{\gamma+\epsilon(1-\gamma)}} H_0 \hat{c} e^{\hat{g}t} \times \left[1 - (1-\hat{c}/c_0)e^{-\frac{[\gamma+\epsilon(1-\gamma)]\rho\hat{c}}{\epsilon}t}\right]^{\frac{\gamma(\epsilon-1)}{\gamma+\epsilon(1-\gamma)}}, \tag{3}$$

$$K(t) = (H_0/\hat{h})(c_0/\hat{c})^{\frac{\epsilon}{\gamma+\epsilon(1-\gamma)}} e^{\hat{g}t}\ {}_2F_1(t), \tag{4}$$

$$H(t) = (c_0/\hat{c})^{\frac{\epsilon}{\gamma+\epsilon(1-\gamma)}} H_0 e^{\hat{g}t} \times \left[1 - (1-\hat{c}/c_0)e^{-\frac{[\gamma+\epsilon(1-\gamma)]\rho\hat{c}}{\epsilon}t}\right]^{\frac{\epsilon}{\gamma+\epsilon(1-\gamma)}}, \tag{5}$$

the long-run growth rate of per capita consumption, output, capital and habits stock is

$$\hat{g} = (A-\theta)/[\gamma + \epsilon(1-\gamma)], \tag{6}$$

the steady-state values of the consumption–habits and habits–capital ratios, $\hat{c}$ and $\hat{h}$, are

$$\hat{c} = 1 + \hat{g}/\rho, \qquad \hat{h} = (\bar{A} - \hat{g})/\hat{c}$$

and the initial consumption–habits ratio, $c_0 = C_0/H_0$, is the (unique) solution to

$$\hat{h}(c_0/\hat{c})^{-\frac{\epsilon}{\gamma+\epsilon(1-\gamma)}} = h_0\, {}_2F_1(0), \tag{7}$$

where $h_0 = H_0/K_0$, and ${}_2F_1(t)$ is the Gauss hypergeometric function

$${}_2F_1(t) = {}_2F_1\left(a, b, b+1, ze^{-\frac{\rho[\gamma+\epsilon(1-\gamma)]\hat{c}}{\epsilon}t}\right), \tag{8}$$

$$a = \gamma(1-\epsilon)/[\epsilon + \gamma(1-\epsilon)],$$
$$b = \epsilon\hat{h}/\{[\gamma+\epsilon(1-\gamma)]\rho\}, \tag{9}$$
$$z = 1 - \hat{c}/c_0,$$

if and only if parameter values are so that $A > \theta > (1-\epsilon)(1-\gamma)A$.

Proof. Similar to Hiraguchi (2012, Theorem 1). □

3. Transition dynamics

3.1. Analytical results

First, we study how the initial (jumpable) consumption–habits ratio changes as the initial (predetermined) habits–capital ratio does. The proof is in the Appendix.

Lemma 2. (i) *The solution c_0 to Eq. (7) satisfies that $dc_0/dh_0 < 0$, with $c_0 > \hat{c}$ if $h_0 < \hat{h}$, $c_0 < \hat{c}$ if $h_0 > \hat{h}$, and $c_0 = \hat{c}$ if $h_0 = \hat{h}$.*
(ii) *Furthermore, if $h_0 \to 0$ then $c_0 \to +\infty$, and if $h_0 \to +\infty$ then $c_0 \to 0$.*

The following results study the non-monotonicity of the time paths of consumption, habits and capital. Let g_x denote the growth rate of the variable x; i.e., $g_x = \dot{x}/x$.

Proposition 3. *The equilibrium path of consumption is non-monotonic if and only if* (i) *$\epsilon > 1$ and $c_0 < 1 - (A-\theta)/[(\epsilon-1)\gamma\rho] < \hat{c}$, or* (ii) *$\epsilon < 1$ and $c_0 > 1 + (A-\theta)/[(1-\epsilon)\gamma\rho] > \hat{c}$.*

Proof. Differentiating the consumption growth rate, $g_C(t) = \dot{C}(t)/C(t)$, it is immediate that $\text{sign}[dg_C(t)/dt] = \text{sign}[(\hat{c}-c_0)(\epsilon-1)]$. Hence, as $g_C(t)$ converges monotonically to $\hat{g} > 0$, a necessary and sufficient condition for non-monotonicity of the consumption path is $g_C(0) = [A - \theta + (\epsilon-1)(c_0-1)\gamma\rho]/\epsilon < 0$. Now, the result follows. □

[1] The original version of this paper provided a proof of this result. More than one year after submitting it, Hiraguchi (2012) independently presented an explicit solution of the 'standard' AK endogenous growth model with multiplicative external habits using hypergeometric functions. However, he did not analyze the transition dynamics of the model so, in particular, he did not notice the non-monotonic dynamics of the variables in level in spite of the monotonicity of their ratios. To save space, we refer to Hiraguchi (2012) for a proof of this result, though our original proof for the general model is available upon request.