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Distribution-sensitivity of rank-dependent poverty measures☆

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ABSTRACT

We propose a criterion to rank poverty measures on the basis of distribution-sensitivity. The criterion compares reactions to 'lossy' transfers among the poor. We focus on the class of rank-dependent poverty measures and provide distribution-sensitivity rankings of the poverty gap ratio, the first and second Sen measures, the Thon measure, the Shorrocks measure, and the Thon, Kakwani and S-Gini classes of measures. Moreover, we discuss the relationship between the proposed criterion and two alternative distribution-sensitivity criteria based on the Arrow–Pratt theory of risk aversion. Finally, we provide an empirically tractable necessary and sufficient condition for unanimous poverty rankings by all continuous and replication invariant rank-dependent poverty measures that exhibit a predetermined minimum degree of distribution-sensitivity.

1. Introduction

Since the seminal contributions of Watts (1968) and Sen (1976), it is broadly accepted that a poverty measure should satisfy the transfer principle.[1] This principle demands that an income transfer from a better off poor individual to a worse off poor individual decreases poverty. The degree of distribution-sensitivity imposed by the transfer principle is minimal, as no sacrifice of mean income is required in return for the distributional improvement produced by the transfer.

Poverty measures typically go beyond the minimal degree of distribution-sensitivity embodied by the transfer principle. Moreover, poverty measures differ considerably in the *extent* to which they go beyond. That is, some poverty measures generally tolerate greater sacrifices of mean income in return for a given distributional improvement than others. Although the literature has always regarded the degree of distribution-sensitivity as an important distinguishing factor, a formal definition of distribution-sensitivity comparisons of poverty measures has long been lacking.[2]

Zheng (2000a) was the first to provide a solid theoretical foundation for comparisons of distribution-sensitivity. Zheng's distribution-sensitivity criterion, which is based on the Arrow–Pratt theory of risk aversion, applies only to the class of *subgroup-consistent* poverty measures. This is a severe limitation: Zheng's criterion does not allow comparisons within the other major class of *rank-dependent* poverty measures, which includes prominent measures proposed by Sen (1976), Thon (1979, 1983), Kakwani (1980), Shorrocks (1995) and others. Zheng (2000a, p. 135) acknowledges this limitation of his criterion and calls for an extension that would allow such comparisons.

We propose a new criterion of distribution-sensitivity that applies to all poverty measures. Following Atkinson (1973) and Okun (1975), we use 'lossy' transfers to gauge the importance that a poverty measure attributes to distribution relative to mean income. Consider a transfer in which the better off poor individual foregoes an amount $a + \ell$, whereas the worse off poor individual receives only an amount a. The amount ℓ is lost in the process of redistribution—Okun (1975, pp. 96–100) discusses potential real-world sources of loss, including administrative costs, reduced work effort, and distorted saving and investment decisions. A transfer without a loss ($\ell = 0$) decreases poverty by virtue of the transfer principle. The largest loss ℓ a poverty measure tolerates without indicating increased poverty quantifies the extent to which the measure goes beyond the minimal degree of distribution-sensitivity expressed by the transfer principle. According to the *lossy transfer criterion* of distribution-sensitivity, a poverty measure P is at least as distribution-sensitive as a poverty measure R if P accepts each lossy transfer that R accepts.

The first objective of the paper is to apply the proposed lossy transfer criterion to the class of rank-dependent poverty measures.

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[1] See Chakravarty (2009, Chapter 2), Lambert (2001, Chapter 6), Seidl (1988) and Zheng (1997) for surveys of the literature on poverty measurement.

[2] The literature frequently uses the term 'poverty aversion' as a synonym for distribution-sensitivity. However, this is somewhat of a misnomer because it is not the 'dislike toward poverty' that is the issue. See also Zheng (2000a, pp. 120–121).

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We demonstrate that the criterion translates into an easy-to-check condition on the weights of rank-dependent poverty measures. It turns out that the distribution-sensitivity criterion conclusively ranks several of the prominent members of the class. For example, the Sen measure in general allows larger losses in transferring from better off to worse off than the Thon measure.

To gain further insight into the lossy transfer criterion, we compare it to two criteria that both derive from the Arrow–Pratt theory of risk aversion: a criterion based on lossy equalizations (creating complete equality among the poor at the cost of mean income) and Zheng's criterion. First, we show that the three criteria coincide on the class of subgroup-consistent poverty measures. Hence, the lossy transfer criterion and the lossy equalization criterion can be interpreted as two alternative generalizations of Zheng's criterion. Second, we demonstrate that the lossy equalization criterion is stronger than the lossy transfer criterion for the class of rank-dependent poverty measures. That is, the lossy equalization criterion ranks each pair of rank-dependent poverty measures that is ranked by the lossy transfer criterion, but in addition ranks pairs that are not ranked by the latter. We will argue, however, that the lossy transfer criterion better captures the idea of distribution-sensitivity than the lossy equalization criterion.

The second objective of the paper is to explore unanimous rankings of income distributions by a class of poverty measures exhibiting a predetermined minimum degree of distribution-sensitivity. This application of the idea of distribution-sensitivity was suggested by Zheng (2000a), who was in turn inspired by Meyer's (1977) work on minimum risk averse unanimity rankings in the theory of choice under risk. Zheng examines the idea for the class of subgroup-consistent poverty measures. We present the complementary analysis for the class of continuous and replication invariant rank-dependent poverty measures. We provide an empirically tractable necessary and sufficient condition for two income distributions to be ranked by the minimum distribution-sensitivity unanimity poverty ranking. Moreover, we argue that this unanimity ranking is a useful extension of the standard concept of censored generalized Lorenz dominance.

The next section introduces notation and presents a brief overview of the rank-dependent poverty measures proposed in the literature. Section 3 defines the main distribution-sensitivity criterion based on lossy transfers and applies it to obtain rankings of the prominent rank-dependent poverty measures. In Section 4, we consider the lossy equalization criterion and Zheng's criterion and discuss the relationships with the lossy transfer criterion. Finally, Section 5 discusses minimum distribution-sensitivity unanimity poverty rankings. All proofs are relegated to the Appendix.

2. Rank-dependent poverty measures

The income of individual i is a positive real number x_i and the income distribution for a population of n individuals is a vector $x = (x_1, x_2, \ldots, x_n)$ in $\mathbb{R}^n_{++}$. The set of income distributions for one or more individuals is $X = \bigcup_{n\in\mathbb{N}} \mathbb{R}^n_{++}$. For each income distribution x in X, the incomes are ordered such that $x_1 \le x_2 \le \cdots \le x_n$. The poverty line is an income level z in $\mathbb{R}^n_{++}$. An individual i is poor if $x_i < z$ and non-poor if $x_i \ge z$. For an income distribution x in X, we write n_x for the number of individuals and q_x for the number of poor individuals. We drop the subscripts in n_x and q_x whenever this does not lead to confusion. For an income distribution x in X, we write $\hat{x}$ for the censored income distribution $(x_1, x_2, \ldots, x_q, z, z, \ldots, z)$. A poverty measure is a function $P : X \to \mathbb{R}$. The value $P(x)$ is to be interpreted as the poverty level associated with income distribution x in X.

A poverty measure P is rank-dependent if, for each income distribution x in X,

$$P(x) = \sum_{i=1}^{q} w_i(q, n)\,\frac{z - x_i}{z}, \tag{1}$$

where $w_1(q, n) \ge w_2(q, n) \ge \cdots \ge w_q(q, n) > 0$.[3] If the weights decrease strictly with the income position, then the poverty measure satisfies the transfer principle. This principle requires that an income transfer from a better off poor individual to a worse off poor individual decreases poverty.[4]

Table 1
Rank-dependent poverty measures.

Measure	$w_i(q, n)$	$\frac{w_i(q,n)}{w_j(q,n)}$
Poverty gap ratio	$\frac{1}{n}$	1
Sen	$\frac{2(q+1-i)}{(q+1)n}$	$\frac{q+1-i}{q+1-j}$
Second Sen	$\frac{2(q+0.5-i)}{qn}$	$\frac{q+0.5-i}{q+0.5-j}$
Thon	$\frac{2(n+1-i)}{(n+1)n}$	$\frac{n+1-i}{n+1-j}$
Shorrocks	$\frac{2(n+0.5-i)}{n^2}$	$\frac{n+0.5-i}{n+0.5-j}$
Kakwani class	$\frac{q(q+1-i)^\kappa}{n\sum_{i=1}^{q} i^\kappa}, \kappa \ge 0$	$\left(\frac{q+1-i}{q+1-j}\right)^\kappa$
Thon class	$\frac{\tau n+2-2i}{(\tau-1)n^2}, \tau \ge 2$	$\frac{\tau n+2-2i}{\tau n+2-2j}$
S-Gini class	$\left(\frac{n+1-i}{n}\right)^\sigma - \left(\frac{n-i}{n}\right)^\sigma, \sigma \ge 1$	$\frac{(n+1-i)^\sigma-(n-i)^\sigma}{(n+1-j)^\sigma-(n-j)^\sigma}$

Table 1 presents rank-dependent poverty measures that have been proposed in the literature (first two columns). The poverty gap ratio equates the weights of all income positions and hence disregards distribution. The two Sen (1976) measures were introduced with the explicit goal of bringing in the distributional concern in the form of the transfer principle. These two measures, as all subsequent measures in the table, feature weights that strictly decrease with the income position. The Thon (1979) and Shorrocks (1995) measures condemn a transfer from a worse off to a better off poor individual even if the recipient crosses the poverty line, a property violated by the two Sen measures. The Kakwani (1980) class of poverty measures is based on the idea that a transfer should have a greater impact on poverty as it takes place further down in the income distribution. The Thon (1983) class, in contrast, reflects the idea that transfers should have the same impact irrespective of their location. For the Kakwani class, the poverty gap ratio is obtained for $\kappa = 0$ and the Sen measure for $\kappa = 1$. The class of S-Gini poverty measures results from the combination of the S-Gini social welfare measure (Donaldson and Weymark, 1980) and Chakravarty's (1983, p. 79) welfare-based poverty measure. The poverty gap ratio is obtained for $\sigma = 1$ and the Shorrocks measure for $\sigma = 2$ (see Chakravarty, 1997). A class that further generalizes the S-Gini class is considered in Section 5.

3. A distribution-sensitivity criterion

Consider an income transfer from a better off poor individual to a worse off poor individual. If the amount the donor gives equals

[3] The form in Eq. (1) is relative. Multiplication by z gives the absolute form. The results in this paper are not affected by the choice between these two forms.

[4] Formally, a poverty measure P satisfies the transfer principle if, for each income distribution x in X and each $a > 0$, we have that $x_i < x_i+a \le x_j-a < x_j < z$ implies $P(x) > P(x_1, x_2, \ldots, x_i + a, \ldots, x_j - a, \ldots, x_n)$. Incidentally, note that each rank-dependent poverty measure satisfies the standard properties of focus (increasing the income of a non-poor individual does not affect poverty) and monotonicity (decreasing the income of a poor individual increases poverty).