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Optimal dividend policy with random interest rates

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ABSTRACT

Several recent papers have studied the impact of macroeconomic shocks on the financial policies of firms. However, they only consider the case where these macroeconomic shocks affect the profitability of firms but not the financial markets conditions. We study the polar case where the profitability of firms is stationary, but interest rates and issuance costs are governed by an exogenous Markov chain. We characterize the optimal dividend policy and show that these two macroeconomic factors have opposing effects: all things being equal, firms distribute more dividends when interest rates are high and less when issuing costs are high.

1. Introduction

Since Jeanblanc-Picqué and Shiryaev (1995) and Radner and Shepp (1996), a sizable literature has investigated the optimal dividend policy problem for a company that is not allowed to issue new securities or obtain a new loan from a bank. The default time is then defined as the first time when the cash reserves of the company fall below zero. In that case, the optimal dividend policy is simple and natural: distribute dividends whenever the level of cash reserves exceeds a certain threshold that depends on the characteristics (drift, volatility) of the cash flow process and the interest rate demanded by shareholders.

An interesting extension of this problem is to investigate how the optimal dividend policy is modified when the profitability of the firm changes over time, due in particular to business cycle fluctuations. As clearly shown by Gertler and Hubbard (1993) and more recently by Hackbarth et al. (2006), macroeconomic conditions have indeed a strong impact on dividend policies through the changes in the profitability of individual firms that they induce. For example, Cadenillas and Sotomayor (2008) solve for the optimal dividend policy when the drift and the volatility of the cash flow process are governed by a Markov chain representing macroeconomic fluctuations. Bolton et al. (2011) study more generally the impact of changing macroeconomic conditions on both the financial and investment policies of the firms. However, Gertler and Hubbard (1993) also show that macroeconomic conditions directly influence payments to shareholders, even independently of each firm's specific earnings performance. Two natural channels for this influence are the fluctuations in interest rates demanded by investors, and the conditions of the credit market.

The purpose of this paper is to examine how these macroeconomic fluctuations influence the dividend policies of firms, even in the absence of fluctuations in their earning processes. In other words, we study the polar case to the one considered in the literature: the drift and volatility of the cash flow process are constant, but the interest rate demanded by investors follows a Markov chain. In a recent paper, Jiang and Pistorius (2012) consider a similar case where both the profitability of the firm and the discount factor follow a Markov chain. Our paper differs in two respects from Jiang and Pistorius (2012). First we adopt direct approach: we solve the couple of ODEs that characterize the solution by using standard numerical techniques. By contrast, Jiang and Pistorius (2012) characterize the solution as the fixed point of a functional operator and find this solution by an iterative algorithm. The second, and more important, difference between our paper and Jiang and Pistorius (2012) is that we allow the firm to issue new securities. This possibility is not only realistic, but it also leads to two non-trivial consequences: the ranking of optimal dividend thresholds across the two states is not always the same; issuance may be optimal even when cash reserves are still positive. This shows

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that introducing possibilities of new issuances is not just a trivial extension, but gives rise to new, economically relevant, results.

Section 2 presents the model and the mathematical characterization of the optimal dividend policy (Theorem 2.1). Section 3 establishes several important properties of the value function. In Section 3.1, we show that the value function remains concave in the level of cash holdings, even when interest rates are stochastic (Theorem 3.1). The concavity of the value function allows us to prove that it is a smooth solution of the corresponding dynamic programming equation (Proposition 3.1). In particular, it satisfies the *smooth fit* condition which is crucial in the numerical resolution of these types of problems. These mathematical results are necessary to establish an important economic result in Section 3.3: the firm will distribute dividends more often when interest rates are high than when they are low (Proposition 3.2). This result comes from the fact that the opportunity cost of cash reserves is higher when the interest rates demanded by investors are high. However, it does not fit well with the empirical evidence, given that firms actually tend to distribute less dividends during recessions (when interest rates are high) than during booms (when interest rates are low) even when the changes in firms' individual profitability are corrected for Gertler and Hubbard (1993). This suggests that other macroeconomic factors, such as the size of frictions on financial markets, must play a role. This is why Section 4 introduces the possibility for the firm to make new equity issuances. When the cost of these new issues (a proxy for the size of financial frictions) is substantially higher during recessions than during booms, the ranking of dividend thresholds is reversed, and firms now distribute more dividends during booms than during recessions.

We also provide the numerical evidence for the above conclusions. In particular, in Section 3.4, the sensitivity analysis with respect to the mean and volatility of the cash flow rate and jump rates between two different interest rate regimes are presented. The mathematical results proved in Section 3 are also essential in constructing and verifying the numerical algorithm. Section 4 gives several numerical illustrations of the case where new equity issuance is possible.

2. Model and characterization of the solution

Uncertainty is described by $(\Omega, \mathbb{F}, \mathbb{P})$, a filtered probability space satisfying the usual assumptions.[1] Let B_t be a one-dimensional standard Brownian motion and $\{i_t\}_{t\geq 0}$ be a simple stationary Markov process taking values in $\{0, 1\}$ with jump rates $\lambda(0), \lambda(1) > 0$. The process $\{i_t\}_{t\geq 0}$ is assumed to be independent from the Brownian motion. The state $i = 0$ is the "good" economic state with a lower interest rate $r_\ell > 0$ and $i = 1$ corresponds to the "bad" state with interest rate $r_h > r_\ell > 0$. We also set $\lambda_\ell := \lambda(0)$ and $\lambda_h := \lambda(1)$.

The cash holdings $\{X_t\}_{t\geq 0}$ of the company follow a diffusion process. Positive dividend payments of any size can be made at any time. However, the cash level is supposed to remain nonnegative at all times. This constraint clearly places a restriction on the possible dividend size. Mathematically,

$$dX_t = \mu dt + \sigma dB_t - dL_t, \tag{2.1}$$

where $\mu, \sigma > 0$ are given constants and the *cumulative dividend payments* L_t is an adaptive, nondecreasing, càdlàg process with $L_{0^-} = 0$. Given a dividend process L and an initial condition $x \in \mathbb{R}$, let $X^{x,L}$ be the unique solution of (2.1), i.e.,

$$X_t^{x,L} = x + \mu t + \sigma B_t - L_t, \quad t \geq 0.$$

Let $\theta = \theta^{x,L}$ be the first exit time of $X^{x,L}$ from the positive real line. This variable θ defines the time of bankruptcy. In what follows we will suppress the dependence on x, L unless this dependence is important. We say that L is *admissible* at the initial level x, if $X_t^{x,L} \geq 0$, for all time $t \in [0, \theta^{x,L}]$ with probability one. We denote the set of all admissible strategies by $\mathcal{A}(x)$. We note that, the admissibility condition is relevant only at the exit time. Indeed, we only require that the cash level process does not jump into negative real line. In economic terms, this means that shareholders can never distribute themselves a dividend that exceeds the cash holdings of the firm. Hence, $X_\theta^{x,L} = 0$. Since the dividend policy beyond the exit time is irrelevant, we simply set $L_t = L_\theta$ for all $t \geq \theta$. In particular, $L_\theta - L_{\theta^-} = X_{\theta^-}$.

The optimal dividend problem is to maximize

$$J(x, i, L) := \mathbb{E}\left[\int_0^\theta \Lambda_t \, dL_t \,\middle|\, i_0 = i, X_{0^-} = x\right],$$
$$\Lambda_t := \exp\left(-\int_0^t r(i_u)du\right).$$

The *value function* is then defined by

$$v(x, i) := \sup_{\mathcal{A}(x)} J(x, i, L), \qquad v_\ell(x) := v(x, 0),$$
$$v_h(x) := v(x, 1). \tag{2.2}$$

The case of a deterministic (and constant) interest rate (i.e., $r_\ell = r_h$) is exactly the problem studied by Jeanblanc-Picqué and Shiryaev (1995) and Radner and Shepp (1996). For future reference, we record that the value function with constant interest rate r is given by

$$V(x, r) := \sup_{L\in\mathcal{A}(x)} \mathbb{E}\left[\int_0^\theta e^{-rt} dL_t | X_{0^-} = x\right]. \tag{2.3}$$

Then, it is clear that

$$0 \leq V(x, r_h) \leq v_h(x) \leq v_\ell(x) \leq V(x, r_l), \quad \forall x \in \mathbb{R}^+. \tag{2.4}$$

2.1. Characterization of the solution

Our main mathematical result is the following characterization of the value function. The existence part of this theorem will be proved in several steps in the subsequent sections. The uniqueness follows from the classical verification argument (see for instance Fleming and Soner (1993)). This characterization of the value function and the properties of the thresholds are essential in our numerical experiments. Indeed, the numerical algorithm is based on these properties. Moreover, the uniqueness ensures that the computed functions are in fact equal to the value function.

Theorem 2.1. *The value function $v = (v(\cdot, 0), v(\cdot, 1)) = (v_h, v_\ell)$ is the unique concave function satisfying the following conditions:*

- *$v_\ell, v_h \in C^2[0, \infty)$ and $v_l(0) = v_h(0) = 0$;*
- *$v'(x, i) \geq 1$ for all x;*
- *For every $x > 0$ and $i \in \{0, 1\}$, $r(i)v(x, i) - \mathcal{L}v(x, i) \geq 0$, where*

$$\mathcal{L}v(x, i) := \mu v'(x, i) + \frac{\sigma^2}{2} v''(x, i) + \lambda(i)[v(x, i+1) - v(x, i)]; \tag{2.5}$$

  with the convention that $i + 1$ denotes the other state than i;
- *there are two positive thresholds $0 < x_h := x(1)$ and $x_\ell := x(0) < \infty$ such that*

$$v'(x, i) = 1, \quad \text{for } x \geq x(i), \quad \text{and} \quad r(i)v(x, i) - \mathcal{L}v(x, i) = 0, \quad \text{for } x \leq x(i).$$

[1] See Karatzas and Shreve (1991) for details.