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An Intuitive Discussion on the Ideal Ramp Filter in Computed Tomography (I)

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Abstract—In X-ray computed tomography (CT), the ideal ramp filter is a generalized function defined by the inverse Fourier transform. Similar to Dirac's discussion on the delta function, we present an intuitive discussion on the ideal ramp filter. With this concise discussion, one obtains a better understanding of the filter backprojection algorithm (FBP) and can easily construct new practical filters.

Keywords—X-ray CT, Image reconstruction, Ideal ramp filter, Generalized function, Filter design.

1. INTRODUCTION

X-ray computed tomography (CT) has undergone tremendous advancement over the last few years. The most popular approach for image reconstruction remains the filtered back-projection (FBP) [1,2] because of its computational advantages. Theoretically, each parallel-beam projection, $p(t,\theta)$, is convolved with the ideal ramp filter, $h(t)$, to obtain a filtered projection,

$$\tilde{p}(t,\theta) = p(t,\theta) * h(t), \tag{1}$$

where $h(t)$ is a generalized function defined by the inverse Fourier transform,

$$h(t) = \int_{-\infty}^{+\infty} |\omega| \exp(i2\pi\omega t)\, d\omega. \tag{2}$$

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In literature, there coexist two inconsistent ways to deal with generalized functions, either mathematically nonrigorous but intuitive, or mathematically rigorous but abstruse [3]. For instance, in dealing with the Dirac delta function, physicists [4] and engineers [5] treat it as a point charge at the origin or an impulse at a reference moment, denoted by

$$\delta(x) = \begin{cases} 0, & x \neq 0, \\ \infty, & x = 0, \end{cases} \tag{3}$$

$$\int_{-\infty}^{+\infty} \delta(x)\, dx = 1. \tag{4}$$

Mathematicians, on the other hand, think it is logically unacceptable that a function is zero almost everywhere but has area of unity. Therefore, they rigorously define Dirac delta as *a continuous linear functional mapping an infinitely differentiable compact support function* $\varphi(x)$ *to the number* $\varphi(0)$ [3,6–8], denoted by

$$\langle \delta, \varphi \rangle = \varphi(0). \tag{5}$$

Certainly we have no reason to reject the mathematical rigorousness, but we really appreciate the intuitive definition and properties of Dirac delta, which brings us a "much better impression" [3, Line 13, p. 11] and enables us to easily solve real problems in many fields, even in theoretical physics [4].

Although the theory on the singular generalized functions can be found in many mathematical monographs [3,6–8], CT engineers wish an intuitive discussion on the ideal ramp filter to better understand CT algorithm and design various filters. In this paper, we try to give a self-contained intuitive discussion on the expression and properties of the ideal ramp filter and point out how to reconstruct new practical filters. Elementary calculus is enough to understand this paper. Readers interested in mathematical rigorousness are referred to suitable reference at the end of the paper.

2. AN INTUITIVE EXPRESSION OF IDEAL RAMP FILTER

Similar to Dirac's definition for delta function, in this section, we will derive an intuitive definition for the ideal ramp filter, which is the base for the following sections.

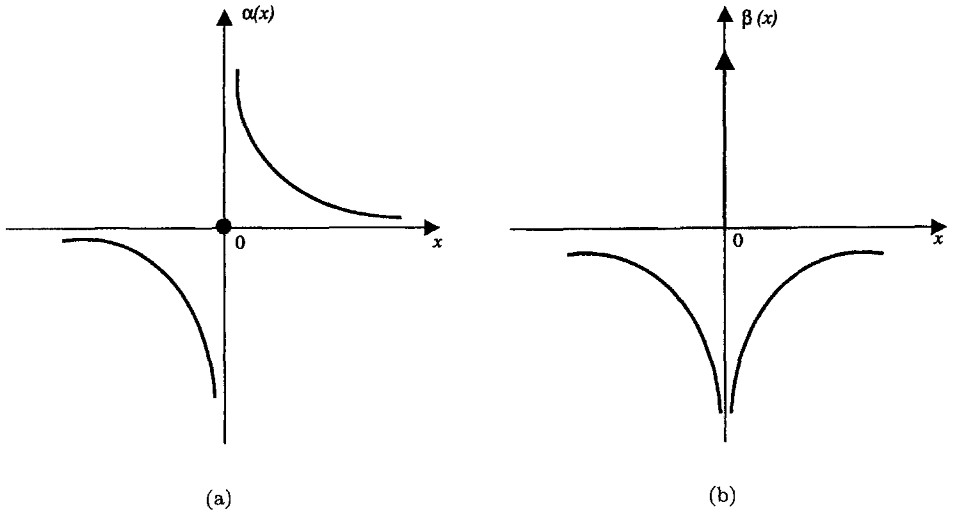

Figure 1. Illustration of (a) $\alpha(x)$, and (b) its derivative function $\beta(x)$.