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Computer-Based Algorithms for Multiple Criteria and Multiple Constraint Level Integer Linear Programming

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Abstract—This paper investigates algorithm development and implementation for multicriteria and multiconstraint level (MC^2) integer linear programming problems. MC^2 linear programming is an extension of linear programming (LP) and multiple criteria (MC) linear programming and a promising computer-aided decision technique in many applications. Here, we present two of the most recent techniques, the MC^2 branch-and-partition algorithm and the MC^2 branch-and-bound algorithm, to solve MC^2 integer linear programs. We describe the design and implementation of a C++ software library for these approaches, and then conduct a comparison study in terms of computational efficiency and complexity through a series of empirical tests.

Keywords—Multicriteria and multiconstraint level linear programming, Branch-and-bound algorithm, Branch-and-partition algorithm, C++ syntax, Integer solutions.

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1. INTRODUCTION

Linear programming (LP) is formulated with a single criterion (objective) and a single (fixed) resource availability level (right-hand side) [1,2]. For the past five decades, linear programming has been widely applied in real-world decision-making problems. However, like any great advance, linear programming is not a perfect tool. For example, if a decision problem involves multiple conflicting criteria, such as maximizing profit while minimizing production cost, linear programming may have limitations in effectively addressing possible tradeoffs among the criteria. This shortcoming has been overcome by mathematical models known as multiple criteria (MC) linear programming [3–5]. MC linear programming improves the value of linear programming by changing its single criterion to multiple criteria.

Although MC models already improve decision-making processes with conflicting criteria, there are many situations where the decisions depend upon multiple constraint levels. In such situations while the managers seek simultaneous criteria, they want to satisfy all decision makers' preference or suggestions for resource availability, some of which conflict with each other. For example, consider a profit-making software firm, where in addition to making money, the company wants to grow, to develop its products and its employees, to provide job security to its workers, and to serve the community. Besides the multicriteria, each product of the company is decided by a group of people, the president of the company, the project manager, the finance manager, etc. Some of these criteria and resource available levels complement each other and others are in direct conflict. Add this to legal, social, and ethical considerations and the dynamic resource availability, the system of criteria and constraints begins to look quite complex. Multiple-criteria and multiconstraint level (MC^2) linear programming has been proposed to overcome the decision problems with both conflicting criteria and resource availabilities represented by the preferences of decision makers [4,6]. This field has become an important research topic in operations research/management science, not only because of the multicriteria and multiconstraint level nature of most real-world decision problems, but also because it opens up many questions to researchers and practitioners.

Seiford and Yu [7] observed that because the criteria coefficients of a primal MC linear programming are the constraint levels of its dual program (by duality theory), the multiple-constraint levels could be built within the structure of the linear system like multiple criteria. This led to the concept, formulation, and development of MC^2 linear programming. This model is supported by both the mathematical structure of the linear system and real applications. In addition to the extension of MC linear programming and linear programming, MC^2 linear programming explicitly expresses multiple (discrete) recourse availability levels. In other words, a single fixed feasible set of linear programming is replaced by several flexible and feasible sets.

An MC^2 linear programming problem can be formulated as

$$\begin{aligned} \text{Max} \quad & \lambda^t Cx, \\ \text{Subject to} \quad & Ax = D\gamma, \\ & x \geq 0, \end{aligned} \tag{1}$$

where $C \in R^{qxn}$, $A \in R^{mxn}$, and $D \in R^{mxp}$ are matrices, $x \in R^n$ are decision variables, $\lambda \in R^q$ is called the criteria parameter, and $\gamma \in R^p$ is called the constraint level parameter. Both vectors (λ, γ) are assumed unknown. The above MC^2 problem has q criteria (objectives) and p constraint levels (resource availability levels). If the constraint level parameter vector γ is known, then the MC^2 problem reduces to an MC linear programming problem. In addition, if the criteria parameter λ vector is known, the problem reduces to a linear program.

Based on the above framework, Shi and Lee [8] proposed a formulation and algorithm for MC^2 binary (or 0-1 variable) linear programming. A general model of MC^2 integer linear programming was explored by Li and Shi [9]. Although a branch-and-partition algorithm for solving MC^2 integer linear programming was demonstrated in the paper, the computer implementation of