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Permanence and Global Attractivity of Delay Diffusive Prey-Predator Systems with the Michaelis-Menten Functional Response

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Abstract—In this paper, the author investigates two-species nonautonomous delay diffusive prey-predator models with Michaelis-Menten functional response. Sufficient conditions are derived for permanence of the species. Further, the author also establishs the sufficient conditions that guarantee the arbitrary positive periodic solution of the systems is global attractive. © 2005 Elsevier Ltd. All rights reserved.

Keywords—Delay, Diffusion, Functional response, Ultimately-bounded domain, Permanence, Global attractivity.

1. INTRODUCTION

The spatial element in population biology is important in understanding the dynamics of ecological systems. For general ordinary differential equations without time delay, Takeuchi [1] showed global stability of diffusive cooperative system under appropriate conditions and Takeuchi [2] discussed the persistence of two species model. In 1999, Huo, Zhang and Chen [3] investigated the dynamics with both diffusion process and Michaelis-Menten functional response. On the other hand, time delays occur so often in nature, a number of models in ecology can be formulated as systems of differential equations with time delays. One of the most important problems for this type of system is to analyze the effect of time delays on the stability of the system. From the literature on ecological models with time delays, we have known that for some systems [4], the stability switches many times and the systems will eventually become unstable when time delays increase, while for other systems, for example [5,6], there will be no change in uniform persistence

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or permanence of the systems when the time delays vary. Uniform persistence or permanence concerning the long time survival of a population is a more important concept of stability from the viewpoint of mathematical ecology.

However, the diffusion and time delays occur simultaneously so often, in almost every true situation. In 1994, Li and He [7] considered the oscillation of delay logistic equation with diffusion, and in 1996, Yang, Chen and Chen [8] investigated the single-species delay diffusive models with nonlinear growth rates. However, they considered only single-species models.

In this paper, I consider the following more realistic two-species prey-predator models with Michaelis-Menten functional response,

$$\begin{aligned} x_1'(t) &= x_1(t)\left[r_1(t) - a_{11}(t)x_1(t) + a_{12}(t)x_1(t-\tau) - \frac{\alpha y(t)}{1+\beta x_1(t)}\right] + d_1(t)\left(x_2(t) - x_1(t)\right), \\ x_2'(t) &= x_2(t)\left[r_2(t) - a_{22}(t)x_2(t) + a_{23}(t)x_2(t-\tau)\right] + d_2(t)\left(x_1(t) - x_2(t)\right), \\ y'(t) &= y(t)\left[r_3(t) + e(t)\frac{\alpha x_1(t)}{1+\beta x_1(t)} - a_{33}(t)y(t) + a_{34}(t)y(t-\tau)\right], \end{aligned} \tag{1.1}$$

and

$$\begin{aligned} x_1'(t) &= x_1(t)\left[r_1(t) - a_{11}(t)x_1(t) - a_{12}(t)x_1(t-\tau) - \frac{\alpha y(t)}{1+\beta x_1(t)}\right] + d_1(t)\left(x_2(t) - x_1(t)\right), \\ x_2'(t) &= x_2(t)\left[r_2(t) - a_{22}(t)x_2(t) - a_{23}(t)x_2(t-\tau)\right] + d_2(t)\left(x_1(t) - x_2(t)\right), \\ y'(t) &= y(t)\left[r_3(t) + e(t)\frac{\alpha x_1(t)}{1+\beta x_1(t)} - a_{33}(t)y(t) - a_{34}(t)y(t-\tau)\right], \end{aligned} \tag{1.2}$$

where $x_i(t)$ is the density of species x in patch i $(i = 1, 2)$, $y(t)$ is the density of species y in patch 1,

$$\begin{gathered} x_i(t) = \varphi_i(t) \ge 0 \qquad (i = 1, 2), \\ y(t) = \varphi_3(t) \ge 0, \qquad \varphi_i(0) > 0 \quad (i = 1, 2, 3), \\ t \in [-\tau, 0], \qquad \tau \ge 0, \qquad \varphi_i \in C([-\tau, 0], R), r_i(t), a_{ij}(t), e(t) \end{gathered}$$

are continuous functions which have positive upper bound and positive lower bound,

$$d_i(t) \qquad (i = 1, 2),$$

is nonnegative bounded continuous function, α and β are positive constants. In systems (1.1) and (1.2), I assume that the growth rate of the first species in each patch are dependent on their past histories and that the growth rate of the second species are dependent on their past histories. Furthermore, instead of a linear inhibition, I assume that a nonlinear inhibition in the form is the Michaelis-Menten (or Holling type II) functional response which is proposed by May [9].

The difference between the two models is that the signs of their time delay terms are opposite. The first model considers two species that experience a positive time delay (the sign of the time delay term is positive), which is a delay due to gestation (see [10]). Thinking of the probability of the deleterious effect of the time delay on a species growth rate (see [11]), the second model involves a passive time delay (the sign of the time delay term is negative). By employing the Lyapunov-Razumikin technique, I establish sufficient conditions that ensure permanence and also establish sufficient conditions that guarantee the arbitrary positive periodic solution of systems is global attractive.

The organization of this paper is as follows. In next section, some notations and lemmas are given. In Sections 3 and 4, I employ differential inequalities and Lyapunov-Razumikhin type theorems to obtain an ultimately bounded domain and establish sufficient conditions that ensure the arbitrary positive periodic solution of systems is global attractive.