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Continuous Optimization

Non-differentiable higher-order symmetric duality in mathematical programming with generalized invexity

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Abstract

A pair of non-differentiable higher-order symmetric dual model in mathematical programming is formulated. The weak and strong duality theorems are established under higher-order-invexity assumption. Symmetric minimax mixed integer primal and dual problems are also investigated.

Keywords: Higher-order symmetric duality; Non-differentiable programming; Duality theorems; Higher-order-invexity

1. Introduction

Higher-order duality in non-linear programming has been studied in last few years by many researchers [6,11,12,17,20]. Mangasarian [6] formulated a class of higher-order dual problems for non-linear programming problems. Mond and Zhang [17] obtained duality results for various higher-order dual programming problems under higher-order invexity assumptions. Recently, under more general invexity-type assumptions, such as higher-order type-I, higher-order pseudo-type-I and higher-order quasi-type-I conditions, Mishra and Rueda [11,12] gave various duality results, which included Mangasarian [6] higher-order duality and Mond–Weir [16] higher-order duality as special cases.

Symmetric duality in non-linear programming problem was first introduced by Dorn [3], who defined a mathematical programming problem and its dual to be symmetric, if the dual of the dual is the primal problem. Later, Dantzig et al. [2] and Mond [14] formulated a pair of symmetric dual programs for scalar function $f(x,y)$ that is convex in the first variable and concave in the second variable. Mond and Weir [16] gave another pair of symmetric dual problem under weaker convexity assumptions imposed on $f(x,y)$.

Mond [13], Hou and Yang [5], Mishra [7–10] and Yang et al. [19] studied second-order duality for non-linear programs. Motivated by Mishra and Rueda [11,12], Mond and Schechter [15] and Zhang [20], we use

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the ideas of Hou and Yang [5] as well as Yang et al. [19] to formulate a pair of higher-order symmetric dual model. We establish higher-order weak, strong and self-duality theorems under higher-order-invexity assumptions. Furthermore, symmetric minimax mixed integer primal and dual problems are also investigated.

2. Preliminaries

Throughout this paper, denote by R^n the n-dimensional Euclidean space and R^n_+ the non-negative orthant of R^n, respectively.

Let C be a compact convex set in R^n. The support function of C is defined by

$$s(x|C) = \max\{x^{\mathrm{T}}y : y \in C\}.$$

A support function, being convex and everywhere finite, has a subdifferential [18], that is, there exists $z \in R^n$ such that

$$s(y|C) \geqslant s(x|C) + z^{\mathrm{T}}(y - x), \quad \forall y \in C.$$

The subdifferential of $s(x|C)$ is given by

$$\partial s(x|C) = \{z \in C : z^{\mathrm{T}}x = s(x|C)\}.$$

For any set $D \subset R^n$, the normal cone to D at a point $x \in D$ is defined by

$$N_D(x) = \{y \in R^n : y^{\mathrm{T}}(z - x) \leqslant 0, \forall z \in D\}.$$

It is obvious that for a compact convex set C, $y \in N_C(x)$ if and only if $s(y|C) = x^{\mathrm{T}}y$, or equivalently, $x \in \partial s(y|C)$.

For a real-valued twice differentiable function $g(x,y)$ defined on an open set in $R^n \times R^m$, denote by $\nabla_x g(\bar{x},\bar{y})$ the gradient of g with respect to x at $(\bar{x},\bar{y})$, $\nabla_{xx} g(\bar{x},\bar{y})$ the Hessian matrix with respect to x at $(\bar{x},\bar{y})$. Similarly, $\nabla_y g(\bar{x},\bar{y})$, $\nabla_{xy} g(\bar{x},\bar{y})$ and $\nabla_{yy} g(\bar{x},\bar{y})$ are also defined.

Definition 1. A function $f : X \to R$ is said to be *higher-order-invex* at $u \in X$ with respect to $\eta : X \times X \to R^n$ and $h : X \times R^n \to R$, if for all $(x,p) \in X \times R^n$,

$$f(x) - f(u) \geqslant \eta(x,u)^{\mathrm{T}}[\nabla_u f(u) + \nabla_p h(u,p)] + h(u,p) - p^{\mathrm{T}}\nabla_p h(u,p).$$

Definition 2. A function $f : X \to R$ is said to be *higher-order-pseudo-invex* at $u \in X$ with respect to $\eta : X \times X \to R^n$ and $h : X \times R^n \to R$, if for all $(x,p) \in X \times R^n$,

$$\eta(x,u)^{\mathrm{T}}[\nabla_u f(u) + \nabla_p h(u,p)] \geqslant 0 \Rightarrow f(x) - f(u) - h(u,p) + p^{\mathrm{T}}\nabla_p h(u,p) \geqslant 0.$$

3. Higher-order symmetric duality

We consider the following higher-order symmetric duality problem.

$$\text{(SHP)} \quad \min \quad f(x,y) + s(x|C) - y^{\mathrm{T}}z + h(x,y,p) - p^{\mathrm{T}}\nabla_p h(x,y,p)$$

$$\text{subject to} \quad \nabla_y f(x,y) - z + \nabla_p h(x,y,p) \leqslant 0, \tag{1}$$

$$y^{\mathrm{T}}[\nabla_y f(x,y) - z + \nabla_p h(x,y,p)] \geqslant 0, \tag{2}$$

$$z \in D, \tag{3}$$