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Discrete Optimization

Beam search heuristic to solve stochastic integer problems under probabilistic constraints

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Abstract

This paper proposes a Beam Search heuristic strategy to solve stochastic integer programming problems under probabilistic constraints. Beam Search is an adaptation of the classical Branch and Bound method in which at any level of the search tree only the most promising nodes are kept for further exploration, whereas the remaining are pruned out permanently. The proposed algorithm has been compared with the Branch and Bound method. The numerical results collected on the probabilistic set covering problem show that the Beam Search technique is very efficient and appears to be a promising tool to solve difficult stochastic integer problems under probabilistic constraints.

Keywords: Stochastic programming; Combinatorial optimization; Beam search heuristic

1. Introduction

This paper addresses stochastic programming problems under probabilistic constraints where both the decision variables and the random variables are restricted to be integer. In particular, the class of problems considered throughout can be mathematically formulated as follows:

$$\text{(SIPC)} \quad \min \quad c^{\mathrm{T}}x \tag{1}$$

$$Ax \geqslant b \tag{2}$$

$$\mathbb{P}\{Tx \geqslant \xi\} \geqslant p \tag{3}$$

$$x \geqslant 0 \quad \text{integer.} \tag{4}$$

Here T is a $m\times n$ integer matrix, A is a $q\times n$ matrix, $c, x\in\mathbb{R}^n$, $b\in\mathbb{R}^q$, ξ a m-dimensional random vector integer-valued and $\mathbb{P}$ denotes probability. Constraints (2) and (3) represent the deterministic and stochastic side of the problem, respectively. In particular, (3) are probabilistic constraints which ensure the satisfaction of the stochastic constraints $Tx \geqslant \xi$ with a prescribed probability level

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$p \in (0,1)$. In the following, we shall use $\mathbb{Z}$ and $|\cdot|_1$ to denote the set of integers and the ℓ_1 norm, respectively. Furthermore, the inequality "$\geqslant$" for vectors will be understood coordinate-wise.

SIPC problems arise in a broad spectrum of applications in which integer and combinatorial optimization problems are formulated in the presence of uncertainty and a "reliable solution" is required. Routing and location are two classical examples of applications where probabilistically constrained integer formulations have been applied to define robust strategic plans. The interested readers are referred to the survey papers of [9,13] and to the references therein.

As one might expect, the difficulty in addressing SIPC problems is twofold. First, SIPC problems have a combinatorial nature. Second, (3) are *joint* probabilistic constraints and no assumption on the independence on the components ξ_i, $i=1,\dots,m$, of the random vector ξ, is imposed. To the best of our knowledge, all the probabilistic models proposed in the papers referred above either include individual chance constraints (i.e. probabilistic constraints individually imposed on all the inequalities) or assume independence of the components of the random vector. In both cases, equivalent deterministic formulations of the probabilistic constraints can be easily derived. In particular, the chance constraints

$$\mathbb{P}\{T^i x \geqslant \xi_i\} \geqslant p_i, \quad i = 1, \dots, m$$

can be replaced by the deterministic constraints:

$$T^i x \geqslant F_i^-(p_i), \quad i = 1, \dots, m,$$

where T^i denotes the ith row of the matrix T and F_i^- is the p_i-quantile of the distribution function of ξ_i (see [17]).

In the case of joint probabilistic constraints with independent random components, Eq. (3) can be replaced by

$$\sum_{i=1}^{m} \ln(F_i(T^i x)) \geqslant \ln p,$$

and specific reformulation can be then derived for the case of integer random variables as shown in [7].

The previous assumptions are rather restrictive and often not adequate to represent real-life applications. Nevertheless, the derivation of deterministic equivalent formulations for joint probabilistic constraints requires the generation of the set of p-efficient points of the probability distribution function. The cardinality of this set is typically high even for moderate size of the random vector, amplifying the complexity associated with the solution of a single deterministic integer optimization problem.

The inherent complexity of the SIPC problems poses the crucial problem to design efficient solution approaches. In this respect the literature is rather scarce. Dentcheva et al. proposed in [7] a cone generation method based on the convexification of a deterministic formulation of the problem. A Branch and Bound method embedding efficient strategies for the determination of lower and upper bound values was proposed in [3].

Both the approaches mentioned above belong to the class of exact solution methods. Because of the complexity of the SIPC problems, the applicability of these methods is typically limited to problems of small and medium sizes. As in deterministic setting, practical-sized instances may be solved rather effectively by resorting to heuristic solution approaches. This paper represents a contribution in this direction. In particular, we propose a heuristic solution approach based on a Beam Search strategy. This is an adaptation of the Branch and Bound method proposed in [3] in which at any level of the search tree only the promising nodes are kept for further exploration, whereas the remaining are pruned off permanently.

The rest of the paper is organized as follows. In Section 2 we briefly recall the Branch and Bound method on which the heuristic approach is based on. Section 3 is devoted to the description of the Beam Search heuristic strategy for the solution of SIPC problems. The efficiency of the approach is tested on the probabilistic set covering problem formulated in [2]. The presentation and discussion of the numerical results is reported in Section 4. The paper ends with some concluding remarks.

2. The Branch and Bound method

The Branch and Bound method for SIPC problems operates on a deterministic equivalent formu-