Available online at www.sciencedirect.com

SCIENCE DIRECT®

ELSEVIER

European Journal of Operational Research 165 (2005) 359–374

EUROPEAN JOURNAL OF OPERATIONAL RESEARCH

www.elsevier.com/locate/dsw

New computational results for the discrete time/cost trade-off problem with time-switch constraints

Mario Vanhoucke

Department of Management Information, Operations Management and Technology Policy,
Faculty of Economics and Business Administration, Ghent University Hoveniersberg 24, B-9000 Ghent, Belgium
Operations and Technology Management Center, Vlerick Leuven Gent Management School, Reep 1, B-9000 Ghent, Belgium

Received 1 October 2002; accepted 1 March 2003
Available online 28 May 2004

Abstract

Recently, time-switch constraints have been introduced in the literature by Yang and Chen [Eur. J. Operat. Res. 120 (2000) 603]. Basically, these constraints impose a specified starting time on the project activities and force them to be inactive during specified time periods. This type of constraints have been incorporated into the well-known discrete time/cost trade-off problem in order to cope with day, night and weekend shifts.

In this paper, we propose a new branch-and-bound algorithm which outperforms the previous one by Vanhoucke et al. [J. Operat. Res. Soc. 53 (2002) 1]. The procedure makes use of a lower bound calculation for the discrete time/cost trade-off problem (without time-switch constraints). The procedure has been coded in Visual C++, version 6.0 under Windows 2000 and has been validated on a randomly generated problem set.

Keywords: Project management; CPM; Time/cost trade-off problem; Time-switch constraints

1. Introduction

Time/cost trade-offs in project networks have been the subject of intensive research since the development of the critical path method (CPM) in the late 1950s. Time/cost behaviour in a project activity basically describes the trade-off between the duration of the activity and its amount of non-renewable resources (e.g. money) committed to it. It is generally accepted that the trade-off follows a non-increasing pattern, i.e. expediting an activity is possible by allocating more resources (i.e. at a larger cost) to it. However, due to its complexity (the problem is known to be NP hard, see De et al., 1997), the problem has been studied under a number of different assumptions.

The early time/cost trade-off models assumed the direct activity cost functions to be *linear* non-increasing functions. The objective was to determine the activity durations and to schedule the activities in order to minimize the project costs, i.e. the sum of the direct activity and the time-dependent indirect project costs, within a specified project deadline. Therefore, the activity costs are a function of the activity durations, which are

---

E-mail addresses: mario.vanhoucke@ugent.be, mario.vanhoucke@vlerick.be (M. Vanhoucke).

0377-2217/$ - see front matter © 2004 Elsevier B.V. All rights reserved.
doi:10.1016/j.ejor.2004.04.007

bounded from below (crash duration) and from above (normal duration). Solution procedures for the linear case are proposed by Kelley and Walker (1959), Fulkerson (1961), Kelley (1961), Ford and Fulkerson (1962), Siemens (1971), Goyal (1975) and Elmaghraby and Salem (1981). Several other forms of activity cost functions have been studied, such as *concave* (Falk and Horowitz, 1972), *convex* (Lamberson and Hocking, 1970; Kapur, 1973; Siemens and Gooding, 1975; Elmaghraby and Salem, 1980a,b) or even *general* continuous activity cost functions (Moder et al., 1983).

As research efforts progressed and practical needs arose, researchers began to focus on the development of procedures for solving the *discrete* version of the problem. This discrete time/cost trade-off problem (further abbreviated as the *DTCTP*) occurs when the duration of project activities is a discrete, non-increasing function of the amount of a single non-renewable resource committed to them. It involves the selection of a set of execution modes (the time–cost tuples for each activity) in order to achieve a certain objective. In the literature, the problem objective has been divided into three parts. The so-called *deadline* problem (problem 1, $T|cpm, \delta_n, disc, mu|av$ following the classification scheme of Herroelen et al. (1999)) aims at minimizing the total cost of the project while meeting a given deadline whereas the *budget* problem (problem 1, $T|cpm, disc, mu|C_{\max}$) involves minimizing the project duration without exceeding a given budget. A third objective is to construct the complete and efficient time/cost profile over the set of feasible project durations (problem 1, $T|cpm, disc, mu|curve$). Research papers have been written by Crowston and Thompson (1967), Crowston (1970), Robinson (1975) Billstein and Radermacher (1977), Wiest and Levy (1977), Hindelang and Muth (1979), Patterson and Harvey (1979), Bianco and Speranza (1990), Vercellis (1990), Elmaghraby and Kamburowski (1992), De et al. (1995, 1997), Demeulemeester et al. (1996, 1998), Skutella (1998) and Akkan et al. (2000a,b). In an attempt to tighten the bridge with the current practice, Vanhoucke et al. (2002) have extended this discrete problem type with so-called time-switch constraints, as introduced by Yang and Chen (2000).

Although the first research endeavors stem from more than 50 years ago, the problem still is very present. Indeed, next to the famous project examples (e.g., the tunnel between France and the United Kingdom), production managers also focus on the development of unique products, such as the development of a new automation system, the installation of a new software program, and many others. Moreover, due to the increasing competition, many firms are obliged to accomplish tasks that do not fit neatly into business-as-usual. Project management fits very well into this new philosophy, due to its unique characteristics and its known and limited duration. Finally, in our current environment of time-based competition, expediting activities is a matter of course, which defends the use of time/cost trade-offs at the activity level.

In this paper, we focus on the discrete time/cost trade-off problem with time-switch constraints (further abbreviated as the *DTCTPTSC*) for which the literature is, to the best of our knowledge, restricted to the paper by Vanhoucke et al. (2002). These constraints impose specific starting times on the project activities and force them to be active and inactive during specific time periods. Consequently, these types of constraints serve well for incorporating day and night shifts. De et al. (1997) have shown that the discrete time/cost trade-off problem is NP-hard under the three objectives mentioned above. Consequently, the DTCTPTSC, which is an extension of the classical DTCTP, is also NP hard. In the following section we briefly review the features of the problem and the branch-and-bound approach of Vanhoucke et al. (2002). In Section 3 we describe our new exact procedure. We also illustrate the procedures using a real-life problem example. A section is reserved on detailed computational results for both the exact procedure and a heuristic one by limiting the allowed CPU-time. The last section draws overall conclusions.

2. Description of the problem

In the sequel of this paper, we assume that a project is represented by an activity-on-the-arc network $G = (N, A)$ where the set of nodes, N,