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Optimal inventory policies for an economic order quantity model with decreasing cost functions

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Abstract

In this paper, three total cost minimization EOQ based inventory problems are modeled and analyzed using geometric programming (GP) techniques. Through GP, optimal solutions for these models are found and sensitivity analysis is performed to investigate the effects of percentage changes in the primal objective function coefficients. The effects on the changes in the optimal order quantity and total cost when different parameters of the problems are changed is also investigated. In addition, a comparative analysis between the total cost minimization models and the basic EOQ model is conducted. By investigating the error in the optimal order quantity and total cost of these models, several interesting economic implications and managerial insights can be observed.

Keywords: Inventory; Geometric programming; EOQ

1. Introduction

This purpose of this work is to extend the classical economic order quantity (EOQ) model to more realistic scenarios. The classic EOQ model assumes constant demand and a fixed purchasing cost. These assumptions do not accurately reflect the time-based competition of today. To this end, three inventory models considering total cost minimization are established and analyzed. The key feature differentiating these models from the basic EOQ model is that the cost per unit exhibits some type of economies of scale. In the EOQ model, the cost per unit is fixed. In the three proposed models, the cost per unit is a power function of the demand per unit time (Model 1), a power function of the order quantity (Model 2), or a power function of both of the demand per unit time and the order quantity (Model 3). In deriving and

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analyzing the optimal solutions, geometric programming (GP) techniques as well as derivative based classical first and second order conditions are used.

GP can be effectively applied to these models to derive the optimal solutions and it gives several advantages over the classical method of calculus. The advantages will be illustrated in finding the optimal solutions for these minimization models and in the managerial implications that can be derived for the optimal policy through bounding and sensitivity analysis.

GP has been very popular in engineering design research since its inception in the early 1960s. Even though GP is an excellent method to solve nonlinear problems, the use of GP in inventory models has been relatively infrequent. Kochenberger [12] was the first to solve the basic EOQ model using GP. In Worrall and Hall [18], GP techniques were utilized to solve an inventory model with multiple items subject to multiple constraints. Cheng [4,5] applied GP to solve modified EOQ models and to perform sensitivity analysis.

There have been numerous publications on EOQ models with fixed cost per unit. Recently, however, several papers relaxed the assumption of the fixed cost per unit for the EOQ models. For example, Lee [14,15] assumed the cost per unit as a function of the order quantity. This assumption means that the production exhibits economies of scale when the order quantity increases. In Cheng [5], Jung and Klein [11], Lee and Kim [16], and Lee et al. [17], the cost per unit was assumed to be a function of the demand per unit time which means that the decision maker employs better equipment and more resources for the production of the product when the demand per unit time increases. Cheng [6] investigated a multiplicative term where the cost per unit is a function of the demand per unit time and the process reliability. This indicates that the cost per unit time is affected by both of the demand per unit time and the process reliability.

In Jung and Klein [11], the total cost minimization model and the profit maximization model were compared and the differences in the optimal order quantity of these two models were investigated, analyzed and discussed. In this paper, we compare three different cost minimization models and investigate the differences in the optimal order quantities and in the optimal total costs. The difference in the optimal order quantity (total cost) of these models indicates the quantity (total cost) that is over-ordered/under-ordered (over-cost/under-cost) due to the error in estimating the cost function of the models. From the comparison, we derive relationships between the optimal solutions by comparing our cost functions without computing the optimal solutions. This means that we can determine optimal inventory policy by estimating the cost functions. We also compare the EOQ model to the minimization models. Since the EOQ model has a fixed unit cost and our models extend the EOQ model by making the unit cost dynamic, we can observe how the EOQ model can be improved.

The remainder of this paper is organized as follows. First, we present assumptions and the three models for total cost minimization. We optimally determine the order quantity for the each model and perform a sensitivity analysis. In the next section, we obtain the optimality results using the first and second order conditions. That is, the changes in the optimal order quantity and total cost according to varied parameters are analyzed to see the effect on inventory policy. Then, we compare and contrast the three cost minimization models as well as the EOQ model to the total cost minimization models to gain managerial insights. Finally, we make concluding remarks and comment on future research areas.

2. Assumptions

We define the following variables and parameters for our models.

D demand per unit time (units/unit time)
Q order quantity (units, decision variable)
C cost per unit (dollar/unit)