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Decision Aiding

Understanding local ignorance and non-specificity within the DS/AHP method of multi-criteria decision making

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Abstract

DS/AHP is a nascent technique for multi-criteria decision-making, inspired by the Analytic Hierarchy Process and whose mathematical foundation is based on the Dempster–Shafer theory of evidence (DST). One aspect, a consequence of the utilisation of DST in DS/AHP is the allowance of a decision maker to make preference judgements on groups of decision alternatives (DAs) over different criteria. In this paper, developments are undertaken on DS/AHP with respect to the measurement of the level of local ignorance in the judgements made, together with the elucidation of an associated measure of non-specificity. Through the evaluation of the limits on these measures, subsequent index values are constructed. These index values are used to introduce the concept of a vagueness plot within DS/AHP. The vagueness plot is shown to aid in gauging the results of interactive changes in the preference judgements made. The index values and the vagueness plot presented enable the notion of the anchoring and adjustment aspect to decision-making in DS/AHP to be investigated.

Keywords: Dempster–Shafer theory; DS/AHP; Ignorance; Non-specificity; Multi-criteria decision making

1. Introduction

Multi-criteria decision-making (MCDM) is concerned with the ranking of decision alternatives (DAs), based on preference judgements made on the DAs over a number of criteria. A variety of techniques have been developed to aid a decision maker (DM) in a MCDM environment. Amongst the most well-known of the MCDM techniques are the Analytic Hierarchy Process (AHP) (Saaty, 1980), and SMART (Von Winterfeldt and Edwards, 1986). Recently, the DS/AHP method of MCDM was introduced in Beynon et al. (2000), which is structurally similar to AHP with a hierarchy of levels of decision-making inherent. However, its mathematical foundation is based on the Dempster–Shafer theory of evidence (DST), introduced in the work of Dempster (1968) and Shafer (1976).

The use of DST in DS/AHP allows a DM to make preference judgements on groups of DAs rather on individual DA or through pairwise comparisons of DAs (as in SMART and AHP).

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In DS/AHP the inclusion or non-inclusion of DAs in identified groups is associated with the notion of ignorance. As such, DS/AHP is a method that should be utilised where there is a presence of ignorance in a MCDM problem. Smets (1991), highlights that ignorance encapsulates incompleteness, imprecision and uncertainty, which here relate to the objective and subjective aspects of the judgement making process. The results from a DS/AHP analysis are levels of preference on groups of DAs (basic probability assignments in DST) as well as a level of concomitant ignorance, collectively called a body of evidence (BOE). DS/AHP enables levels of belief and plausibility to be identified (from a BOE) on the best DA(s) existing within varying sized groups of DAs.

In this paper an investigation is undertaken on the limits of local ignorance and the not previously considered non-specificity in the judgements made by a DM when using DS/AHP. With non-specificity, this relates to the size of the identified groups of DAs, whether a DM has identified a large number of relatively small sized groups of DAs (low non-specificity) or a small number of large sized groups of DAs (high non-specificity). Importantly, the identification of a group of DAs in DS/AHP implies no specific information on the individual DAs contained in the group. Hence the presence of non-specificity, its non-presence would require further assumptions possibly including the 'principle of insufficient reasoning' as described in Beynon et al. (2000). To exposit this novel approach to the internal structuring of the judgements in DS/AHP, informal comparisons are made with results from SMART and AHP analyses.

Through an example, the local ignorance and non-specificity measures are exposited, and associated index values constructed. Moreover, two types of index values are defined, namely unconstrained and constrained index values. Unconstrained index values (for local ignorance and non-specificity) incorporate the range of these measures when limits on the scale values available to be used and the number of DAs identified by a DM are considered. In contrast, the constrained index values include limits on scale values together with retaining the information on the number of DAs and groups of DAs initially identified by a DM. The index values for local ignorance and non-specificity are utilised in the introduction of a vagueness plot.

This vagueness plot attempts to elucidate the level of intensity in the judgements made by a DM. It is shown to allow the results of any changes in the preference judgements made by a DM to be assessed based on the changes on the intensity of the judgement making described by the measures previously defined. An example set of changes in the judgements made in the small example problem elucidate the interpretation a vagueness plot may offer. These developments also allow the association of DS/AHP with the notion of anchoring and adjustment found in the seminal work of Lichtenstein and Slovic (1971) and Tversky and Kahneman (1974). In DS/AHP, the local ignorance and non-specificity index values as the anchor simplify the understanding of the position of the anchor and its limiting values.

The structure of the rest of the paper is as follows: In Section 2, a brief exposition of DST and DS/AHP are given through an example problem, including also comparisons of results with other techniques. In Section 3, the role of anchoring and adjustment is considered within DS/AHP. In Sections 4 and 5, expressions for the unconstrained and constrained indexes associated with local ignorance and non-specificity are exposited respectively. In Section 6, the concept of a vagueness plot is exposited. In Section 7, conclusions are discussed.

2. Exposition of Dempster–Shafer theory and DS/AHP

Dempster–Shafer theory (DST) was introduced in the work of Dempster (1968) and Shafer (1976). Within the wide-ranging field of artificial intelligence, DST has concerned itself with uncertain reasoning (Shafer and Pearl, 1990). More specifically, if the state of knowledge on the uncertainty behaviour does not allow one to specify a probability distribution (see Näther and Wünsche, 2002). As stated in Beynon (2002) the DS/AHP method of MCDM is based around DST as a