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Parametric nonlinear programming approach to fuzzy queues with bulk service

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Abstract

This paper proposes a procedure to construct the membership functions of the performance measures in bulk service queuing systems with the arrival rate and service rate are fuzzy numbers. The basic idea is to transform a fuzzy queue with bulk service to a family of conventional crisp queues with bulk service by applying the α-cut approach. On the basis of α-cut representation and the extension principle, a pair of parametric nonlinear programs is formulated to describe that family of crisp bulk service queues, via which the membership functions of the performance measures are derived. To demonstrate the validity of the proposed procedure, two fuzzy queues often encountered in transportation management are exemplified. Since the performance measures are expressed by membership functions rather than by crisp values, they completely conserve the fuzziness of input information when some data of bulk-service queuing systems are ambiguous. Thus the proposed approach for vague systems can represent the system more accurately, and more information is provided for designing queuing systems in real life. By extending to fuzzy environment, the bulk service queuing models would have wider applications.

Keywords: Fuzzy sets; Queuing; Bulk service; Nonlinear programming

1. Introduction

The single server queuing systems with bulk service have been extensively studied by many researchers like Bailey [1], Bhat [2], Borthakur [3], Chaudhary and Templeton [7], Krishnamoorthy and Ushakumari [12], Medhi [14], Neuts [16], and Sivasamy [18]. Examples in real life are an elevator and a cable car that can take at most a certain number of passengers at a time. Within the context of traditional queuing theory, the interarrival times and service times are required to follow certain probability distributions. However, in many practical applications, the statistical information may be obtained subjectively; i.e., the arrival pattern and service pattern are more suitably described by linguistic terms such as fast, slow, or moderate, rather than by probability distributions. Thus, fuzzy queues [17] are much more realistic than the

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commonly used crisp queues. If the usual crisp queues with bulk service can be extended to fuzzy queues with bulk service, queuing models would have even wider applications.

Buckley [4] investigated elementary multiple-server queuing systems with finite or infinite capacity and source population, in that the arrivals and departures are followed by possibility distributions; in addition, recently with other two scholars, he applied the previous results to a machine serving problem and a queuing decision problem [5]. On the basis of Zadeh's extension principle [20,21], the possibility concept, and fuzzy Markov chains [19], Li and Lee [13] have derived analytical solutions for two fuzzy queues, namely, $M/F/1$ and $FM/FM/1$, where F denotes fuzzy time and FM denotes fuzzified exponential time. One straightforward idea is to apply their approach to the fuzzy bulk service queuing problems. However, as commented by Negi and Lee [15], their approach is very complicated and is generally unsuitable to computational purposes. Furthermore, as commented by Kao et al. [10], for other more complicated queuing systems, Li and Lee's approach is hardly possible to derive analytical results. Therefore, it is very difficult and unsuitable to apply Li and Lee's approach to fuzzy queues with bulk service. Negi and Lee [15] propose the α-cut and two-variable simulation [6] approaches to analyze fuzzy queues. Unfortunately, their approach only provides crisp solutions; in other words, the membership functions of the performance measures are not completely described. If we can derive the membership function of some performance measure, we obtain a more reasonable and realistic performance measure because it maintains the fuzziness of input information that can be used to represent the fuzzy system more accurately. Kao et al. [10], therefore, adopt parametric programming to construct the membership functions of the performance measures for fuzzy queues, and successfully apply to four simple fuzzy queues with one or two fuzzy variables, namely, $M/F/1$, $F/M/1$, $F/F/1$, and $FM/FM/1$. It seems that their approach is applicable to the fuzzy bulk service queues. However, since the fuzzy bulk service queuing systems are much more complicated than the above four fuzzy queues, the solution procedure for the fuzzy bulk service queue is not explicitly known and deserves further investigation.

In this paper we develop a method that is able to provide fuzzy performance measures for bulk service queues with fuzzified exponential arrival rate (i.e., the expected number of arrivals per time period) and service rate (i.e., the expected number of services per time period). The membership functions of performance measures will be derived. The basic idea is to apply the α-cuts and Zadeh's extension principle [20,21] to transform the fuzzy bulk service queue to a family of crisp bulk service queues. As the α value varies, the crisp bulk service queues are then described and solved by the parametric nonlinear programming (NLP) technique. The solutions from the parametric NLP construct the membership functions of interests.

2. Fuzzy queues with bulk service

In the discussion of exponential queuing systems based on classical queuing theory, the arrival rate λ denotes the expected number of arrivals per time period; that is, the time between successive arrivals are independent exponential random variables having mean $1/\lambda$. The service rate μ denotes the expected number of service completions per time unit; i.e., the successive service times are assumed to be independent exponential random variables having mean $1/\mu$. The commonly used performance measures of queuing systems are (1) the system length L, i.e., the expected number of customers in the system; (2) the queue length L_q, i.e., the expected number of customers in the queue; (3) the system waiting time W, i.e., the expected amount of time a customer spends in the system; and (4) the queue waiting time W_q, i.e., the expected amount of time a customer spends waiting in the queue.

Consider a queuing system in which the arrivals of customers occur at a single-server facility as a Poisson process with a fuzzy arrival rate $\tilde{\lambda}$, and the server is able to serve K ($\geqslant 1$) customers at a time. They are served FCFS (first-come-first-served), and there is no waiting capacity. The size of source population is infinite. The batch size for service is not limited to be exactly K; i.e., if there are only less than K (say K_0)