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Discrete Optimization

Single machine scheduling with resource dependent release times and processing times

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Abstract

We consider the single machine scheduling problem with resource dependent release times and processing times, in which both the release times and processing times are strictly linear decreasing functions of the amount of resources consumed. The objective is to minimize the makespan plus the total resource consumption costs. We propose a heuristic algorithm for the general problem by utilizing some derived optimal properties and analyze its performance bound. For some special cases, we propose another heuristic algorithm that achieves a tighter performance bound.

Keywords: Scheduling; Resource dependent release times; Resource dependent processing times; Heuristics; Performance bound

1. Introduction

The scheduling problem with resource dependent processing times has received much research attention in recent years. Studies in this area were initiated by Vickson [15,16] and Van Wassenhove and Baker [14]. A survey of this topic up to 1990 was given by Nowicki and Zdrzalka [11]. During the last decade, some new results on these problems have appeared in the literature. They can be found in [1–3,5,9,12,13,17,18]. The scheduling models cited above all assume that each job is available at the beginning or its release time is constant.

The scheduling problem with resource dependent release times has also received considerable attention of the scheduling research community in recent years. Some research results can be found in [4,6–8,10]. In these scheduling models, the jobs are each assumed to have a fixed processing time.

However, to the best of our knowledge, there seem to exist no papers studying the scheduling problem in which both release times and processing times are resource dependent. Such a scheduling problem commonly arises in the chemical processing industry. Before chemical compounds (jobs) are ready for

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processing, they have to be preheated to reach a temperature threshold below which chemical reactions will not take place. This preheating process consumes resources such as fuel and so a chemical compound is ready earlier for processing if more fuel is consumed to preheat it. On the other hand, the processing time of a chemical compound varies according to the speed of its chemical reaction, which is directly related to the amount of catalysts consumed. Hence, both the job release times and processing times are variable and depend on the amount of resources consumed. The objective of the scheduling problem is to minimize the sum of resource consumption and the makespan, i.e., the total elapsed time to complete all jobs. Such a situation can be modeled as our scheduling problem with resource dependent release times and processing times on a single machine.

This paper is organized as follows. In Section 2, we formulate the problem under study. In Section 3, we derive some properties of an optimal solution. In Section 4, we present a heuristic algorithm for the general problem and analyze its performance bound. In Section 5, we present a heuristic algorithm for some special cases that yields a tighter performance bound. Section 6 concludes with a summary and suggestions for further research.

2. Problem formulation

In this section the single machine scheduling problem is considered under the assumption that both release times and processing times are strictly linear decreasing functions of the amount of resources consumed. Formally, the problem can be formulated as follows.

We consider the problem of scheduling a set $J = \{J_1, \dots, J_n\}$ of n jobs on a single machine. Let π denote a permutation of the jobs in set J and Π the set of all such permutations. All jobs are initially available at time v, but each job may be made available at an earlier time point by consuming extra resources (e.g. fuel) that will incur additional costs. Associated with each job J_i is a processing time p_i and a release time r_i, $i = 1, 2, \dots, n$, where both p_i and r_i depend on the amount of resources consumed. Specifically, $p_i = a_i - x_i$, where a_i is the normal processing time and x_i the amount of processing time compression, $0 \leqslant x_i \leqslant a_i$; $r_i = v - u_i/w$, where u_i is the cost of the resource consumed to advance the availability of J_i to r_i and w the cost per unit reduction of release time, $0 \leqslant u_i \leqslant wv$. For the convenience of theoretically analyzing the studied problem, it should be noted that the case $p_i = 0$ means that the actual processing time of job J_i is so small that it can be neglected in the objective function. Let $x = (x_1, x_2, \dots, x_n)$ denote a processing time compression vector and $r = (r_1, r_2, \dots, r_n)$ a release time vector. Also, let X denote the set of all feasible x and R the set of all feasible r.

For any job J_i, since the amount of resources consumed for its release time reduction u_i, $u_i = w(v - r_i)$, is a decreasing function of the release time r_i, $i = 1, 2, \dots, n$, we assume that all jobs start as early as possible after they are released. For given π, x and r, assuming the job permutation $\pi = (J_1, \dots, J_n)$, the objective function, or total cost, $K(x, r, \pi)$ is defined as

$$K(x,r,\pi) = \max_{1 \leqslant j \leqslant n} \left\{ r_j + \sum_{i=j}^{n} (a_i - x_i) \right\} + \sum_{i=1}^{n} c_i x_i + \sum_{i=1}^{n} w(v - r_i), \tag{1}$$

where c_i, $i = 1, 2, \dots, n$, is the cost per unit processing time reduction. To simplify notation, we assume that both c_i, $0 < c_i < 1$, and w, $0 < w < 1$, are appropriately scaled so that their units are compatible with that of the makespan. It is clear that $\max_{1 \leqslant j \leqslant n} \{r_j + \sum_{i=j}^{n} (a_i - x_i)\}$ is the makespan, $\sum_{i=1}^{n} c_i x_i$ is the total processing time compressing cost, and $\sum_{i=1}^{n} w(v - r_i)$ is the total release time compressing cost. Thus, the optimal objective $K^*(x, r, \pi)$ is

$$K^*(x,r,\pi) = \min_{x \in X, r \in R, \pi \in \Pi} K(x,r,\pi). \tag{2}$$