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Discrete Optimization

A branch and bound algorithm for the robust spanning tree problem with interval data

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Abstract

The robust spanning tree problem is a variation, motivated by telecommunications applications, of the classic minimum spanning tree problem. In the robust spanning tree problem edge costs lie in an interval instead of having a fixed value.

Interval numbers model uncertainty about the exact cost values. A robust spanning tree is a spanning tree whose total cost minimizes the maximum deviation from the optimal spanning tree over all realizations of the edge costs. This robustness concept is formalized in mathematical terms and is used to drive optimization.

In this paper a branch and bound algorithm for the robust spanning tree problem is proposed. The method embeds the extension of some results previously presented in the literature and some new elements, such as a new lower bound and some new reduction rules, all based on the exploitation of some peculiarities of the branching strategy adopted.

Computational results obtained by the algorithm are presented. The technique we propose is up to 210 faster than methods recently appeared in the literature.

Keywords: Branch and bound; Robust optimization; Interval data; Spanning tree problem

1. Introduction

This paper presents a branch and bound algorithm for a robust version of the minimum spanning tree problem where edge costs lie in an interval instead of having a fixed value. Each interval is used to model uncertainty about the real value of the respective cost, which can take any value in the interval, independently from the costs associated with the other edges of the graph.

Adopting the model described above, the classic optimality criterion of the minimum spanning tree problem (where a fixed cost is associated with each edge of the graph) does not apply anymore, and the classic polynomial-time algorithms [9,11] cannot be used. A more complex optimization criterion has then to be adopted. We have chosen the *relative robustness criterion* (see [7]).

The study has practical motivations, and in particular there are some applications in the field of telecommunications. Consider a supervisor

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node in a data network where transmission lines are subject to uncertain delays, that wants to send a control message to all other nodes in the network. The supervisor node generally wants to broadcast the message over a robust spanning tree, in order to have a relatively quick broadcast whatever the situation in the network is (see [3] for a more detailed description of the problem). A second application concerns the design of communication networks where routing delays on edges are uncertain, since they depend on the network traffic. The ideal network guarantees good performance whatever is the real traffic, i.e. a robust spanning tree is desirable (see [7] for more details).

In the literature there are some other studies related to robust versions of the minimum spanning tree problem. Kozina and Perepelista [8] defined an order relation on the set of feasible solutions and generated a Pareto set. Aron and Van Hentenryck [2] proved that the problem is $\mathcal{NP}$-hard. In Yaman et al. [12] a mixed integer programming formulation and a preprocessing technique are presented.

The present paper describes a branch and bound algorithm, based on an adaptation of the method developed in Montemanni et al. [10] for the *robust shortest path problem*. The algorithm incorporates an extension of the preprocessing rules described in [12] and some new concepts which significantly contribute to the efficiency of the method.

Another branch and bound approach to the robust spanning tree problem has been independently developed by Aron and Van Hentenryck (see [1]). From an algorithmic point of view, the method they propose has weaker preprocessing and reduction rules, a less efficient branching strategy and shares the same lower bound.

In Section 2 the robust spanning tree problem with interval data is formally described. Section 3 is devoted to the presentation of the new branch and bound algorithm and its components. Section 4 is dedicated to computational results, while conclusions are presented in Section 5.

2. Problem description

The robust spanning tree problem with interval data is defined on a graph $G = \{V, E\}$, where V is a set of vertices and E is a set of edges. An interval $[l_{ij}, u_{ij}]$, with $0 \leqslant l_{ij} < u_{ij}$, is associated with each edge $\{i, j\} \in E$. The problem is formally described through the following definitions:

Definition 1. A scenario s is a realization of edge costs, i.e. a cost $c_{ij}^s \in [l_{ij}, u_{ij}]$ is fixed $\forall \{i, j\} \in E$.

Definition 2. The robust deviation for a spanning tree t in a scenario s is the difference between the cost of t in s and the cost of the minimum spanning tree in s.

Definition 3. A spanning tree t is said to be a relative robust spanning tree if it has the smallest (among all spanning trees) maximum (among all possible scenarios) robust deviation.

A scenario can be seen as a snapshot of the network situation, while a relative robust spanning tree (*robust spanning tree* for short) is a tree which minimizes the maximum deviation from the optimal spanning tree over all realizations of the edge costs.

The following result (see [12]) is used by the method we propose:

Given a spanning tree t, a scenario s(t) which makes the robust deviation maximum for t is the one where $c_{ij}^{s(t)} = u_{ij}$ $\forall \{i, j\} \in t$ *and* $c_{kh}^{s(t)} = l_{kh}$ $\forall \{k, h\} \in E \setminus t$.

In the remainder of this paper we will refer to the scenario $s(t)$ as the scenario *induced* by tree t. We will also refer to the cost of t in $s(t)$ minus the cost in $s(t)$ of a minimum spanning tree of $s(t)$ as $RCost(t)$, the *robustness cost* of t.

A polynomial-time procedure for the evaluation of the robustness cost of a given spanning tree t arises. It simply works by subtracting the cost of the minimum spanning tree in scenario $s(t)$ [11] from the cost, in the same scenario, of t.

3. The branch and bound algorithm BB-RST

BB-RST, a Branch and Bound algorithm for the Robust Spanning Tree problem with interval data, is presented in this paper. It is an adaptation of an algorithm recently presented in Montemanni et al.