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Algorithm for minimizing weighted earliness penalty in single-machine problem

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Abstract

In this paper, the problem of minimizing the weighted earliness penalty in a single-machine scheduling problem is addressed. For this problem, every job is assumed to be available at time zero and must be completed before or on its deadline. No tardy job is allowed. Each job has its own earliness penalty and deadline. The paper identifies several local optimality conditions for sequencing of adjacent jobs. A heuristic algorithm is developed based on these local optimality conditions. Sample problems are solved and the solutions obtained from the heuristic are compared to solutions obtained from the heuristics developed by Chand and Schneeberger. Also, comparisons are performed between the solutions obtained from the heuristic and the optimal solutions obtained from a mathematical modeling approach for problems involving 10 and 15 jobs. The results show that the developed heuristic produces solutions close to optimal in small size problems, and it also outperforms the Chand and Schneeberger's method.

Keywords: Scheduling; Heuristic; Deadline; Earliness; Tardiness

1. Introduction

This paper considers the single-machine scheduling problem where the objective is to minimize the weighted earliness penalty. For this class of problem, it is assumed that there are N independent jobs to be processed on a single machine. All jobs are available at time zero and must be completed before or on their deadlines. No tardy job is allowed. Each job has its own earliness penalty and deadline. This kind of scheduling problem is found in situations where tardiness cost is extremely high compared to earliness cost or when tardy jobs are prohibited. This class of problem is also found in production situations that involve many customers and the customers generally operate in a JIT environment where there is no tolerance for tardiness. In this problem, if a job is completed earlier than its deadline, then the earliness

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cost is incurred for holding the job until its delivery time. A job cannot be delivered to a customer before or after its deadline. It must be shipped to the customer exactly on its prespecified deadline. There is a linear earliness penalty for holding a job in inventory until its deadline.

An ideal solution to this problem occurs when all jobs are completed exactly on their deadlines. In this case, there is no earliness cost. However, this situation rarely occurs, especially, if the capacity of the shop is tight or the deadlines of all jobs are clustered together in a small interval of time as is typical in a JIT production. To deal with the problem, one may need to schedule some jobs early and incur the earliness cost for those jobs. Therefore, a good schedule of the jobs for such a problem would be to schedule all jobs to minimize the total weighted earliness penalty of early jobs—the objective of this paper.

Scheduling problems involving earliness costs have been extensively studied under various assumptions and objective functions as presented in the review of Baker and Scudder [4]. The problem of minimizing the earliness penalty in a single-machine scheduling problem when the jobs' deadlines were determined by the equal slack (SLK) method was studied by Qi and Tu [8]. Chhajed [6] introduced a problem where N jobs were to be scheduled on a single machine. Jobs must be assigned one of two deadlines, which were given at equal interval. Job tardiness was not allowed. The deadline cost was considered in addition to the earliness cost. In Asano and Ohta [3], not only was the deadline of each job known, but also the ready time was prespecified. They proposed an optimization algorithm using dominance relation for the scheduling problem. Their algorithm was based on the branch and bound approach.

Ahmadi and Bagchi [1] studied the scheduling problem in which the objective was to minimize the total earliness cost subject to meeting deadlines. They showed that the problem was NP-complete and that the problem was equivalent to the problem of minimizing the total waiting time of all jobs subject to release times and common deadline. They developed a branch and bound method to solve the problem.

The problem of minimizing the total weighted earliness penalty, which is also the objective of this paper was first studied by Ahmadi and Bagchi [2]. They developed a branch and bound method based on unequal weights to solve the problem. Chand and Schneeberger [5] also studied the problem with this objective. They studied two types of problems, which they referred to as the *Weighted Earliness Problem* (WE), and the *Constrained Weighted Earliness Problem* (CWE-Problem). In their WE-problem, machine idle time can be inserted, but idle time is not permitted in CWE-problem. They showed that both problems were NP-hard, and that optimal solutions can be obtained by a polynomial time algorithm only in some specified situations. In their paper, they developed a dynamic programming based approach for solving the WE-problem. They also developed two heuristic algorithms for solving the CWE-problem and WE-problem, respectively. The heuristics were a modification of the Smith heuristic [9].

In this paper, the problem studied is the same as that presented in Ahmadi and Bagchi [2] and in WE-problem by Chand and Schneeberger [5]. The methods developed by Ahmadi and Berger [2], and Chand and Schneeberger [5] are based on the exact solution methods, which are not practical for solving large size problems. Chand and Schneeberger [5] do not recommend using their dynamic programming algorithm for solving problems with more than 15 jobs. The only heuristic algorithm, which was presented by Chand and Schneeberger [5], produced good solutions in some specific cases, but the effectiveness of the heuristic was not quite as good in the other cases.

The objective of this study is to develop an effective solution procedure for solving large-scale single-machine scheduling problems with weighted earliness penalty through a heuristic approach. In Section 2, the problem formulation is given. Several local optimality properties for scheduling adjacent jobs on a machine, which form the basis of this heuristic, are presented in Section 3. Section 4 presents the steps of the heuristic algorithm. An application of the heuristic is given in Section 5. Results and analysis of sample problems are discussed in Section 6. Section 7 is the conclusion. The proofs of some properties are provided in Appendix A.