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Pricing non-storable perishable goods by using a purchase restriction: General optimality results

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Abstract

The aim of this paper is to investigate the validity of the optimality results derived from [Eur. J. Oper. Res. 134 (2001) 631] by relaxing the following two assumptions: (a) the firm offers restricted units first and then unrestricted units later at higher price levels, and (b) only one type of product is available during the whole selling process. In the absence of (a), we establish a general optimality theorem, which shows that any optimal policy in [Eur. J. Oper. Res. 134 (2001) 631] remains to be optimal in the class of general policies that allow the restriction to be attached at any price level. For the simultaneous availability issue associated with (b), we demonstrate that there always exists an optimal policy that is sustainable even when all active prices are made available at the same time. These two results assure the relevance of the theoretical model in [Eur. J. Oper. Res. 134 (2001) 631] to current yield management practices and further improve our understanding on the role and the impact of a well-designed purchase restriction on pricing decisions.

Keywords: Optimal pricing; Perishable goods; Purchase restriction

1. Introduction

Li (2001) develops an optimal pricing model for a monopolist, which uses a purchase restriction as a mechanism to segment the demand market for perishable, non-storable products such as airline seats and hotel rooms. It is shown that by properly setting the level of the highest restricted price and rationing the sales at different prices, the monopolist needs to charge no more than three prices to maximize revenue. On the other hand, there are two assumptions that may limit the applicability of his model. The first one is that the firm offers the restricted units first and then the unrestricted units later at higher prices. For ease of reference, throughout this paper, I will name these types of policies as *primary policies*. Even though a primary policy seems to be natural, it does nevertheless limit the firm's choices of possible pricing structures. Another assumption is that the firm offers one kind of price at a time; which means that if the product

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is selling at a restricted price, then the unrestricted units will *not be available*. In the airline case, when both restricted fares and unrestricted fares are offered, both types of fare are available at the same time. So a critic may say that the model in Li (2001) is *inconsistent* with practice; hence the validity of his results may be in doubt. Let us call this *the simultaneous availability issue*.

The aim of this paper is to completely resolve these two outstanding issues. Section 2 establishes some preliminary results on general pricing policies that allow the firm to attach the restriction at any price level. Section 3 formally proves a general optimality theorem, which shows that it is sufficient for the firm to limit its attention to the primary policies in searching for an optimal policy. In Section 4, we establish a simultaneous availability property by showing that there always exists an optimal primary policy that is sustainable even when all active prices are made available. Conclusion and additional discussions follow in the last section.

2. Auxiliary results on general pricing policies

The same model settings will be used as in Li (2001). There is a monopoly firm selling a fixed number of units of a certain product that is perishable and not storable for consumers, such as airline tickets or hotel rooms. The market demand is given by the function $D(P)$, which can be interpreted as the number of consumers whose reservation price is P or higher. $D(P)$ is assumed to be non-increasing and left-continuous. The firm has an option to impose a restriction when a consumer purchases the product. The impact of the restriction on demand is captured by a function $\alpha(P)$, which is interpreted as *the percentage of those consumers with a reservation price P, or higher, who cannot accommodate the restriction*.

To simplify the analysis, as in Wilson (1988) and Li (2001), the (original) demand function $D(P)$ is assumed to be a *step function*. More precisely, there exists a set of observed demand points $\{(P_i, D_i) : i = 1, \ldots, N\}$ such that $P_1 < P_2 < \cdots < P_N$, $D_1 > D_2 > \cdots > D_N$ and [1]

$$D(P) = \begin{cases} D_1 & \text{if } P \leqslant P_1; \\ D_i & \text{if } P \in (P_{i-1}, P_i] \quad \text{for } 2 \leqslant i \leqslant N; \\ 0 & \text{if } P > P_N. \end{cases} \tag{1}$$

Consequently, the demand for the product at price P when the purchase restriction is attached is given by$D_r(P) \equiv (1 - \alpha(P))D(P)$, which is also a step function defined on $\{(P_i, D_{r,i}) : i = 1, \ldots, N\}$ with $D_{r,i} \equiv D_r(P_i) = (1 - \alpha_i)D_i$ where $\alpha_i \equiv \alpha(P_i)$, for $i = 1, \ldots, N$. Throughout this paper, it is assumed that α_i is strictly increasing in i. This monotonicity assumption implies that as the price increases, among these who are willing to buy the product without the restriction, the percentage of them who cannot accommodate the restriction is increasing. As argued in Li (2001), this assumption illustrates the essence of the effectiveness of a purchase restriction.

A few observations should be made before a general pricing policy is defined. First, a general policy should allow the firm to offer restricted and unrestricted units at the same price level in an arbitrary order, in the sense that the firm can sell the unrestricted units first and the restricted units later at the same price, and vice versa. Because of this, it is possible to have more than two allocations at the same price level. For example, the firm may offer some restricted units at price P_1, then some unrestricted units at the same price, then again offer some additional restricted units at this price, and so on. Consequently, the number of initial prices, $P_1, \ldots, P_N$, has to be modified to capture this possibility. Therefore, by looking at the *order* of offered prices together with the option of imposing the restriction, *a sequence of prices* $p_1 \leqslant p_2 \leqslant \cdots \leqslant p_n$ is

[1] There is a minor notational change here from the original model in Li (2001), where the price set is specified by $\{p_1, \ldots, p_n\}$. This change is for the ease of presentation.