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Exit problems in regime-switching models

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Abstract

This paper provides a general framework for pricing of perpetual American and real options in regime-switching Lévy models. In each state of the Markov chain, which determines switches from one Lévy process to another, the payoff stream is a monotone function of the Lévy process labeled by the state. This allows for additional switching within each state of the Markov chain (payoffs can be different in different regions of the real line). The pricing procedure is efficient even if the number of states is large provided the transition rates are not very large w.r.t. the riskless rates. The payoffs and riskless rates may depend on a state. Special cases are stochastic volatility models and models with stochastic interest rate; both must be modeled as finite-state Markov chains. As an application, we solve exit problems for a price-taking firm, and study the dependence of the exit threshold on the interest rate uncertainty.

JEL classification: D81; C61; G31

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1. Introduction

Starting with Hamilton's (1989) paper, there has been a growing body of empirical evidence suggesting that the distribution of returns on a financial asset is driven by a regime-switching process. From an economic perspective, regime-switching behavior captures the changing preferences and beliefs of investors concerning asset prices as the state of financial market changes from a bull market to a bear market, and vice versa. Later, in a number of econometric studies, it was demonstrated that appropriate modeling of equity return distributions requires mean-reversion, as well as jumps in returns and/or volatility—see Bakshi et al. (1997), Chernov et al. (2003) and the bibliography therein. Chernov et al. (2003) note that the key is that the volatility can move violently. This feature can be naturally modeled as regime switching. For regime-switching term structure models, see e.g., Dai et al. (2007).

Usually, in the real options literature, regime switching means fluctuations between different levels of volatility and/or drift in the (geometric) Brownian motion case (this can be viewed as a discretization of more refined stochastic volatility models) or different payoffs or profit flows in different states, which results from possible government interventions such as changes in the tax regime or subsidies. For examples, see Dixit and Pindyck (1996). Regime-switching models become more and more popular in macroeconomics, where policy changes can be naturally captured

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by regime shifts (see for example, Davig, 2004). Sarantis and Piard (2004) use Markov regime-switching modeling to analyze credibility of a number of countries participating in the European Monetary System. In a recent review Guthrie (2006), the impacts of various regulation mechanisms on investment are discussed; regime-switching models can provide an appropriate framework for the theoretical analysis of these mechanisms. Also, regime-switching models are relevant to segmented risk sharing as, for example, in Taub and Chade. (2002) and Chade and Taub (2005). Though in general, empirical evidence suggests that commodity prices behave differently than standard financial asset prices, some commodity prices also exhibit regime-switching dynamics. For example, Casassus et al. (2005) demonstrate that the spot price of a commodity which is primarily used as an input to production, such as oil or copper, is best described by a regime-switching process. There is another important aspect of the real options valuation, which is typically ignored in the literature in order to simplify models, namely, stochastic interest rates (for a discussion about the importance of the interest rate modeling in real option valuation and references, see e.g., Trigeorgis., 1996; Miltersen and Schwartz, 1998; Schulmerich, 2005). Note that regime-switching models with different riskless rates in different states can be interpreted as approximations to models with stochastic interest rates. In many situations, the dynamics of factors exhibit mean reversion (see e.g., Dixit and Pindyck, 1996). The mean-reverting feature can be obtained by introducing appropriate jumps in the price of the underlying at moments of switching.

Finally, it is natural to expect that global warming will lead to regime shifts, and numerous policy interventions can be expected as well. Therefore, a general framework for study of investment decisions under regime switches of various kind is needed. Regime-switching models with a sufficiently rich structure of the modulating Markov chain and different riskless rates and payoffs in different states can reproduce all the effects above with a good accuracy. However, if the number of states is more than two, then the solution is far from being straightforward, and the situation becomes extremely complicated if, instead of the standard model for the profit function: $\pi(K_t, X_t) = G(K_t)\,\mathrm{e}^{X_t} - C_t$, where X_t is the underlying process, we use more general functions. The latter complication is necessary should one want to consider different policy interventions such as taxes, which may be different at different levels of the profit flow (for instance, the negative profit flow is not taxed), price caps or floors, which lead to $\pi(K_t, X_t) = G(K_t)\ \max\{\mathrm{e}^{X_t}, P\} - C_t$ or $\pi(K_t, X_t) = G(K_t)\ \min\{\mathrm{e}^{X_t}, P\} - C_t$, respectively. The latter type of policy interventions was recently discussed as an important stimulus for the development of greener fuels and curbing the gasoline consumption.

This short discussion explains the importance of models which can combine all features listed above; however, the standard methods discussed below rely strongly on the special type of the profit functions, which makes them difficult to adjust for more general profit functions. Hence, it is difficult to use the standard approaches to study policy interventions in the regime-switching environment. Moreover, these methods become analytically untractable if the number of states is greater than two.

The main result of the paper is a general method which can be used to study policy interventions of various kind, in the stochastic environment modulated by a Markov chain with arbitrary number of states; computational realization of the method is efficient even if the number of states is measured in hundreds or even thousands. We introduce the method in the continuous time framework. The method admits a natural modification for discrete time models (cf. Boyarchenko and Levendorskiĭ, 2007). Using the same trick as in Boyarchenko (2004) and Boyarchenko and Levendorskiĭ (2007), it is quite straightforward to reduce the problem of incremental capital expansion to the investment problem studied here.

Different pricing problems in regime-switching models were considered in a number of papers; we give a short review of results for perpetual options and real options as closely related to the subject of the paper. In the majority of publications, Markov-modulated (geometric) Brownian motion models are studied. Guo (2001) obtained closed-form solutions for perpetual lookback options. Pricing of the perpetual American put in the geometric Brownian motion model coupled by a finite-state Markov chain was studied by Guo and Zhang (2004), who obtained explicit results in the case of a two-state Markov chain. Guo et al. (2005) consider a model or irreversible investment with regime shifts. Buffington and Elliott (2002) designed a pricing procedure for the American put with finite time horizon. For further references, see the papers cited above.

The results for switching models with jumps are scarce due to technical difficulties, although the literature on pricing of American options under processes with jumps is fairly extensive by now—see e.g., Alili and Kyprianou (2005), Asmussen et al. (2004), Avram et al. (2004), Boyarchenko and Levendorskiĭ (2002a,b, 2005, 2006), Kyprianou and Pistorius (2003), Levendorskiĭ (2004) and the bibliography therein. Asmussen et al. (2004) studied the perpetual American put in regime-switching Lévy models with phase-type jumps. The technique of the paper relies on the special elegant structure of this class of models and a simple explicit structure of the payoff function. It is not clear how to