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OccBin: A toolkit for solving dynamic models with occasionally binding constraints easily

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A B S T R A C T

The toolkit adapts a first-order perturbation approach and applies it in a piecewise fashion to solve dynamic models with occasionally binding constraints. Our examples include a real business cycle model with a constraint on the level of investment and a New Keynesian model subject to the zero lower bound on nominal interest rates. Compared with a high-quality numerical solution, the piecewise linear perturbation method can adequately capture key properties of the models we consider. A key advantage of the piecewise linear perturbation method is its applicability to models with a large number of state variables.

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1. Introduction

Inequality constraints that bind occasionally arise in a wide array of economic applications. We describe how to adapt a first-order perturbation approach and apply it in a piecewise fashion to handle occasionally binding constraints. To showcase the applicability of our approach, we solve two popular dynamic stochastic models. The first model is an RBC model with limitations on the mobility of factors of production. The second model is a canonical New Keynesian model subject to the zero lower bound on nominal interest rates. As is typical for dynamic models, the models we consider do not have a closed-form analytical solution. In each case, we compare the piecewise linear perturbation solution with a high-quality numerical solution that can be taken to be virtually exact.[2]

Our contribution is twofold. First, we outline an algorithm to obtain a piecewise linear solution. While the individual elements of the algorithm are not original, our recombination simplifies the application of this type of solution to a general class of models.[3] We offer a library of numerical routines, OccBin, that implements the algorithm and is compatible with Dynare, a convenient and popular modeling environment (Adjemian et al., 2011). Second, we present a systematic assessment of the quality of the piecewise linear perturbation method relative to a virtually exact solution, which has not been attempted by others. Because standard perturbation methods only provide a local approximation, they cannot capture

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[2] The virtually exact solution is obtained either by dynamic programming on a very fine lattice for the state variables of the model or by spectral methods, following Christiano and Fisher (2000). In addition to the RBC and New Keynesian models, an online appendix evaluates our solution for a model of consumption choice subject to a constraint on borrowing.

[3] Our approach, including the title of this paper, is inspired by the work by Uhlig (1995) who developed an early toolkit to analyze nonlinear dynamic discrete-time stochastic models without occasionally binding constraints.

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occasionally binding constraints without adaptation. Our analysis builds on an insight that has been used extensively in the literature on the effects of attaining the zero-lower bound on nominal interest rates.[4] That insight is that occasionally binding constraints can be handled as different regimes of the same model. Under one regime, the occasionally binding constraint is slack. Under the other regime, the same constraint is binding. The piecewise linear solution method involves linking the first-order approximation of the model around the same point under each regime. Importantly, the solution that the algorithm produces is not just linear – with two different sets of coefficients depending on whether the occasionally binding constraint is binding or not – but rather, it can be highly nonlinear. The dynamics in one of the two regimes may crucially depend on how long one expects to be in that regime. In turn, how long one expects to be in that regime depends on the state vector. This interaction produces the high nonlinearity.

Our assessment focuses on several aspects of the solution. Following Christiano and Fisher (2000), we compare moments of key variables by reporting mean, standard deviation, and skewness. Following Taylor and Uhlig (1990), we compare plots of stochastic simulations. In addition, we assess the accuracy of the piecewise linear approximation by computing two bounded rationality metrics. The first metric is the Euler equation residual, following Judd (1992). The Euler equation residual quantifies the error in the intertemporal allocation problem using units of consumption. The second metric relies on the broader evaluation of expected utility. Intuitively, the closest approximation to the solution of the model will lead to the highest utility level. The difference in utility implied by two solution methods can also be expressed as a compensating variation in consumption that a utility-maximizing agent would have to be offered in order to continue using the less accurate method. On the basis of these comparisons and assessments, we find that the piecewise linear perturbation method can capture adequately key properties of the models we consider.

We also highlight some limitations of the piecewise linear solution. Namely, just like any linear solution, it discards all information regarding the realization of future shocks. Accordingly, our piecewise linear approach is not able to capture precautionary behavior linked to the possibility that a constraint may become binding in the future, as a result of shocks yet unrealized. However, the piecewise method also inherits some of the key advantages of a first-order perturbation approach. It is computationally fast and applicable to models with a large number of state variables even when the curse of dimensionality renders other higher-quality methods inapplicable.[5] Moreover, our library of numerical routines accepts a model written in a natural way with no meaningful syntax restrictions. Accordingly, application of our algorithm to different models requires only minimal programming.

Section 2 outlines the piecewise linear solution algorithm. Section 3 relates our approach to the literature. Section 4 considers a real business cycle model with a constraint on investment. Section 5 considers a New Keynesian model subject to the zero lower bound on nominal interest rates. Section 6 concludes.

2. The solution algorithm

For clarity of exposition, we confine our attention to a model with only one occasionally binding constraint. Extensions to multiple occasionally binding constraints are implemented in the library of routines.

A model with an occasionally binding constraint is equivalent to one with two regimes. Under one regime, the occasionally binding constraint is slack. Under the other regime, the constraint binds. We linearize the model under each regime around the non-stochastic steady state, although a different point could be chosen. We dub the regime that applies at the point of linearization the "reference" regime, or ($M1$). We dub the other regime "alternative", or ($M2$). It is immaterial whether the occasionally binding constraint is slack at the reference regime or at the alternative regime.

There are two important requirements for the application of our algorithm.

1. The conditions for existence of a rational expectations solution in Blanchard and Kahn (1980) hold at the reference regime.
2. If shocks move the model away from the reference regime to the alternative regime, the model will return to the reference regime in finite time under the assumption that agents expect that no future shocks will occur.[6]

2.1. Definition of a piecewise linear solution

Without loss of generality, when the occasionally binding constraint $g(E_t X_{t+1}, X_t, X_{t-1}) \le 0$ is slack, the linearized system of necessary conditions for an equilibrium under the reference regime can be expressed as

$$\mathcal{A} E_t X_{t+1} + \mathcal{B} X_t + \mathcal{C} X_{t-1} + \mathcal{E}\epsilon_t = 0, \tag{M1}$$

[4] Recent examples of the use of this technique include Jung et al. (2005), Eggertsson and Woodford (2003), Christiano et al. (2011).

[5] The library of routines that accompanies this paper contains additional examples of models that can be solved with a piecewise linear algorithm. One of the examples is the celebrated Smets and Wouters (2007) model, extended to incorporate the zero lower bound on the policy interest rate. As an illustration of the speed of the piecewise linear algorithm, our toolkit solves that model in a fraction of a second.

[6] This restriction might appear draconian, but it is routinely imposed when solving DSGE models with standard first-order perturbation methods. In fact, the linear approximation to the solution could be equivalently characterized as implementing either the rational expectations restrictions or perfect foresight.