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On a two-sector endogenous growth model with quasi-geometric discounting

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ABSTRACT

We study a two-sector endogenous growth model with quasi-geometric discounting in which human capital is the engine of growth. We show that a planning economy welfare-dominates a competitive economy and time-consistent government policy is welfare-improving if the agents are sufficiently patient. The government policy consists of a tax on physical capital income and a subsidy on human capital accumulation. Our results differ from those of existing one-sector models with quasi-geometric discounting in which a competitive economy always outperforms a planning economy and the government's time-consistent tax policies reduce equilibrium welfare.

1. Introduction

Experimental evidence suggests that the discounting of future payoffs is not geometric (Thaler, 1981; Benzion et al., 1989). Salois and Moss (2011) use market asset data and obtain a statistically significant quasi-geometric parameter from an empirical point of view. Economic models with time-inconsistent preferences were initially studied by Strotz (1956), Phelps and Pollak (1968), and Pollak (1968). Laibson (1997) and Barro (1999) reformulate these models by adopting quasi-geometric (quasi-hyperbolic) discounting.

An important study by Krusell et al. (2002), henceforth KKS, introduces quasi-geometric discounting to neoclassical growth models. KKS show how an individual's problem can be solved as a game between the current and future selves. They compare the competitive equilibrium path with the planner's solution path in terms of welfare. Surprisingly, they find that a competitive economy always performs better than a planning economy. They also find that the time-consistent tax policy lowers equilibrium welfare. Their results, however, are based on the assumption of a one-sector economy.

We study a two-sector endogenous growth model with quasi-geometric discounting in which human capital is the growth engine, and check whether the conclusions of KKS still hold. The setup is similar to KKS, except that production of the final good requires physical and human capital. The individual has a unit time endowment, allocated between working and accumulating human capital. We explicitly characterize the competitive economy and the planning economy.

First, we study a three-period example to show that a planning economy always welfare-dominates the competitive economy. We solve the model backward. In the second period, the social planner simply maximizes the utility of the two-period-lived agents subject to resource constraints. In the first period, the planner maximizes the utility of the three-period-lived agents subject to resource constraints during all the three periods *and* the first-order conditions (FOCs) of the second-period problem. Here, the planner cannot ignore the future-period FOCs because the preferences are time inconsistent. Similarly, in the first period, the equilibrium consumer maximizes his utility subject to budget constraints and the second-period FOCs. Since two-period optimization problems are time consistent, the equilibrium FOCs and the planner's FOCs coincide. Thus, the equilibrium allocation satisfies all the constraints that the planner has in the first period, that is, the second-period FOCs and the resource constraints. Because the planner maximizes utility subject to these constraints whereas the equilibrium consumer who takes factor prices as given does not, the planning economy always performs better than the competitive economy as long as the preferences are time inconsistent.

Next, we turn to the infinite horizon model. We compare the equilibrium path with the planner's solution path and show that when the discount factor is close to 1, the planning economy welfare-dominates the competitive economy. We then examine a time-consistent government policy consisting of a physical capital

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tax and a subsidy on human capital accumulation. The physical capital tax rate and human capital subsidy rate are both positive, and, with this policy, the competitive economy coincides with the planning economy. This means that the policy is welfare-improving as long as the agents are sufficiently patient.

In our model, social welfare depends on both the savings rate and working time, but in KKS's model, it depends only on the latter. As in KKS, the equilibrium savings rate is closer to the welfare-maximizing savings rate than the planner's savings rate. Thus, the competitive economy would welfare-dominate the planning economy if the working time in both the economies is the same. However, the difference between the working time and welfare-maximizing working time is greater in the competitive economy than in the planning economy. This is why the planning economy welfare-dominates the competitive economy.

As KKS point out, the planner suffers from a commitment problem in physical capital accumulation when the time horizon is large. When the savings of the future planners are insufficient and the current planner tries to make them accumulate more physical capital, he recognizes that the returns to additional savings diminish from strict concavity of the production function. In a competitive equilibrium, they are constant. Thus, the savings rate of the equilibrium consumer is higher than that of the planner, and the competitive economy becomes closer to the first-best economy. In KKS, the economy has only one sector and then the planning economy is always worse off than the competitive economy. In our paper, however, the economy needs human capital for production in addition to physical capital, and the planner and consumer accumulate human capital in the same manner. Thus, the disadvantage that the planner has over the consumer in physical capital accumulation is weaker than in KKS's model, and the planning economy can result in higher welfare relative to the planning problem, as in a finite-period model.

Quasi-geometric discounting has been applied to many fields of macroeconomics. Laibson (2001) argues that quasi-geometric discounting leads to under-saving. Diamond and Kőszegi (2003) show that workers who resort to quasi-geometric discounting retire early. Schwarz and Sheshinski (2007) and Guo and Caliendo (2014) investigate social security issues. Bisin et al. (forthcoming) study a model of public debt where voters have quasi-geometric discounting. However, these papers are based on either partial equilibrium models or standard one-sector ones. Boulware et al. (2013) investigate a human capital model with quasi-geometric discounting, but they ignore physical capital and do not obtain the equilibrium allocation as in our paper. Few studies have examined non-geometric discounting in a multi-sector economy.[1]

Recent empirical works find that education levels are significantly connected to economic growth (Cohen and Soto, 2007). This suggests the importance of investigating models that explicitly describe human capital accumulation. Two-sector growth models with physical and human capital accumulation were introduced by Uzawa (1968) and Lucas (1988), and numerous extensions have followed these pioneering studies. However, the existing papers use standard constant discounting.

The rest of the paper is organized as follows. Section 2 studies a three-period model. Section 3 investigates an infinite horizon model. Section 4 examines a time-consistent tax policy. Section 5 consider a case where human capital accumulation needs physical capital. Section 6 concludes the paper. The Appendix presents the proofs of various propositions.

[1] Our paper is also close to Gerlagh and Liski (2011), who study private and public investments in a model with quasi-geometric discounting. However, they do not examine the competitive economy.

2. Three-period example

We first study a simple three-period model to see how a planning economy outperforms the competitive economy when the preferences are time inconsistent.

2.1. The environment

We assume three dates, $t = 0, 1, 2$, and a continuum of individuals with a unit measure. The utility function at date 0 is $\ln c_0 + \beta\delta \ln c_1 + \beta\delta^2 \ln c_2$, where c_t is the consumption at date t, δ the discount factor, and β the degree of time inconsistency. An individual's utility at date 1 is $\ln c_1 + \delta\beta \ln c_2$. He has a unit time endowment, and spends l units on producing goods and $1 - l$ units on accumulating human capital. We call *l the working time*. In the following, let the variables with primes be the next-period values. The resource constraint is

$$k' = F(k, lh) - c, \tag{1}$$

where F is the production function, k is the stock of physical capital, h is the stock of human capital, and the term lh represents the efficiency units of labor. The production function is the Cobb–Douglas function $F(K, L) = AK^{\alpha}L^{1-\alpha}$ with $A > 0$ and $\alpha \in (0, 1)$. As in Lucas (1988), the evolution of human capital is governed by

$$h' = B(1 - l)h \quad \text{for } t \geq 0, \tag{2}$$

where $B > 0$. We let $\boldsymbol{x}$ be a vector of the state variables defined as $\boldsymbol{x} = (k, h)$. If we re-write the efficiency units of labor as a function of current and future human capital, $lh = h - h'/B$. Thus, the output can be expressed as $F(k, h - h'/B)$.

Factor markets are competitive. The interest rate is $r = F_K(k, lh) = \alpha Ak^{\alpha-1}(lh)^{1-\alpha}$, and the wage rate is $w = F_L(k, lh) = (1-\alpha)Ak^{\alpha}(lh)^{-\alpha}$, where F_K (F_L) is the marginal product of physical capital (labor). In a competitive economy, the budget constraint is $k' = rk + wlh - c$, while the human capital equation can still be given by Eq. (2).

2.2. Competitive economy

We first obtain the equilibrium allocation. We solve the model backward as Salanié and Treich (2006) do. Let r_t^e and w_t^e denote respectively the equilibrium interest rate and wage rate at date t. In the final period, no benefit arises from accumulating capital. Thus, the individual spends all his time on working, but saves nothing at all. Thus, his choice is $s_2 = 0$ and $l_2 = 1$. At date 1, given the initial state (k_1, h_1), he solves

$$V_1^e(k_1, h_1) = \max_{c_1, l_1, c_2} [\ln c_1 + \beta\delta \ln c_2],$$

$$\text{s.t. } k_2 = r_1^e k_1 + w_1^e l_1 h_1 - c_1, \tag{3}$$

$$0 = r_2^e k_2 + w_2^e h_2 - c_2, \tag{4}$$

$$h_2 = B(1 - l_1)h_1, \tag{5}$$

where V_1^e is his value function at date 1, Eq. (3) is the budget constraint at date 1, and Eq. (4) is the budget constraint at date 2. The FOCs are

$$\frac{c_2}{c_1} = \delta\beta r_2^e, \tag{6}$$

$$r_2^e = \frac{Bw_2^e}{w_1^e}, \tag{7}$$

where Eq. (6) is the Euler equation, and Eq. (7) is the non-arbitrage condition between physical capital and human capital. If the consumer spends one unit of time on studying instead of working, he loses wage income by $w_1^e h_1$ units at date 1. However,