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Additive representation for preferences over menus in finite choice settings

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ABSTRACT

This paper obtains an additive representation for preferences over subsets of a finite set relaxing the two substantive axioms in Kreps (1979) flexibility theorem. The result implies that the lottery structure and assumptions employed by Dekel, Lipman and Rustichini (2001) to identify the subjective state-space do not introduce extraneous restrictions on deterministic choice behavior. This property does not necessarily hold when additional axioms are imposed.

1. Introduction

In a seminal contribution, Kreps (1979) introduced preferences over menus and obtained an additive representation which rationalizes preference for flexibility as subjective uncertainty over future tastes. Subsequent work on temptation and self-control by Gul and Pesendorfer (2001) (henceforth GP) and the generalization of Kreps' original model by Dekel et al. (2001) (henceforth DLR) started a literature axiomatizing numerous types of dynamic choice behavior.[1]

While Kreps (1979) studies a finite choice setting in which a decision maker (henceforth DM) first chooses a menu of deterministic alternatives and then selects one of the alternatives contained in that menu, GP and DLR obtain their representations by introducing lotteries (*i.e.* probability distributions over the alternatives) and considering preferences over menus of those lotteries.

However, as noted by Olszewski (2011), most of the examples (if not all) in this literature consist of deterministic finite choice situations in which lotteries play no role. This is not surprising since some of the most often discussed behavioral phenomena, such as temptation and self-control, are conceptually unrelated to lotteries (see Gul and Pesendorfer, 2005 and Nehring, 2006). The lottery structure is essential for the vast majority of existing results, though.[2]

For example, it is well-known that Kreps' additive representation can involve different sets of second-stage preferences. In other words, it is typically not possible to pin down the behavior associated with the DM's subjective states knowing only how she ranks menus ex-ante. Nevertheless, by introducing lotteries and focusing on DMs who are ex-post expected utility maximizers, DLR are able to essentially identify the state-space and get uniqueness of ex-post behavior. Because of this result, even if there are no actual lotteries for the DM to choose among, we may still want to adopt DLR's framework and interpret the DMs preferences as a thought experiment. But, how are we constraining the observable choice behavior of the DM if we do so?

With this question in mind, the present paper obtains a general additive representation for Kreps' framework. Besides being of intrinsic interest, we use this result to investigate the restrictions that standard axioms for the lottery setting impose on preferences over menus of deterministic alternatives (*i.e.* degenerate lotteries). Since our representation relaxes Kreps' two substantive axioms by allowing for "negative" states, we conclude that the additivity per se has no behavioral content. In this way, we formally verify that the basic assumptions introduced by DLR do not surreptitiously rule out behavior which would be allowed if lotteries were not

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[1] Some examples include Sarver (2008) on regret, Ergin and Sarver (2010) on contemplation costs, Dillenberger and Sadowski (2012) on guilt, Kopylov (2012) on perfectionism and Ortoleva (2013) on thinking aversion.

[2] Of course, there are scenarios involving both temptation and lotteries in a fundamental way. We merely contend that there are many applications for which deterministic choice behavior is the natural object of interest and the introduction of lotteries serves only as a technical device.

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available. In other words, the linear structure employed by DLR to identify the state-space does not sacrifice generality regarding finite deterministic choices.

The lack of restrictions is hardly surprising, given the nature of the DLR assumptions. Nevertheless, imposing additional axioms in the DLR framework might constrain deterministic choices beyond what these new axioms imply if considered on their own. As we will see in Section 2, assumptions such as independence and continuity, despite lacking behavioral content without lotteries, can significantly modify the impact of other axioms in the finite setting in ways which are hard to predict a priori. For example, further assuming "lottery set monotonicity" (*i.e.* that bigger menus of lotteries are always weakly better than smaller ones) restricts the induced preferences over menus of deterministic alternatives to satisfy not only the non-lottery version of this type of monotonicity but also an independent axiom calibrating indifferences called "ordinal submodularity".

The rest of the paper is organized as follows. Section 2 contains the main results and it is divided into three subsections. Section 2.1 presents the basic representation theorem for preferences over menus of deterministic alternatives and explores the effects of imposing additional axioms in the finite setting. Section 2.2 develops the link with the lottery framework and establishes that the assumptions made by DLR do not introduce extraneous restrictions on deterministic choice behavior. Section 2.3 demonstrates that extraneous restrictions might in fact arise if additional axioms are imposed. Section 3 discusses three related topics: consistent collections of second-stage preferences in the finite setting, the role of the ordinal submodularity axiom in avoiding preference for commitment, and further connections with the literature on temptation and self-control. Finally, Appendix A presents a self-contained description of the conjugate Möbius transform (the main mathematical tool in the proof of Proposition 1), while Appendix B includes the proofs omitted from the main text.

2. Main results

2.1. Representation without lotteries

Let X be a non-empty finite set, define $\mathcal{M}$ to be the set of all non-empty subsets of X and let $\succsim$ be a generic binary relation on $\mathcal{M}$ (with $\succ$ and $\sim$ standing for its asymmetric and symmetric parts, respectively). The binary relation $\succsim$ is *represented* by $U \in \mathbb{R}^{\mathcal{M}}$ if $x \succsim y$ is equivalent to $U(x) \geq U(y)$. $\succsim$ is a *preference* if it is complete and transitive.

Kreps (1979) uses this simple setting to model a two-stage decision process. In the first stage, the DM chooses a menu $A \in \mathcal{M}$. In the second stage, she chooses an alternative $x \in A$ from the previously chosen menu. Kreps' main theorem establishes that a preference $\succsim$ on $\mathcal{M}$ satisfies *set monotonicity* ($A \supseteq B$ implies $A \succsim B$) and *ordinal submodularity* ($A \sim A \cup B$ implies $A \cup C \sim A \cup B \cup C$) if and only if it admits a representation $U \in \mathbb{R}^{\mathcal{M}}$ of the form

$$U(A) = \sum_{s \in S} \max_{x \in A} u(x, s),$$

where S is a finite set and $u \in \mathbb{R}^{X \times S}$. The elements of S are usually interpreted as "subjective states", indexing the possible second-stage preferences encoded in u.

Note that every state $s \in S$ in Kreps' representation is "positive", in the sense that a higher second-stage indirect utility $\max_{x \in A} u(x, s)$ induces higher first-stage utility $U(A)$. However, one can also rationalize "negative" states, for instance, in the case of a DM who may succumb to temptation and deviate from ex-ante better alternatives. Gul and Pesendorfer (2005) study the *set betweenness* axiom ($A \succsim B$ implies $A \succsim A \cup B \succsim B$), which is consistent with this type of behavior.

As it turns out, it is possible to relax both set monotonicity and ordinal submodularity if we allow negative states in the representation. To formalize the idea, we say that function $U \in \mathbb{R}^{\mathcal{M}}$ is *additive* if there exist finite sets S_1 and S_2 and a function $u \in \mathbb{R}^{X \times (S_1 \cup S_2)}$ such that

$$U(A) = \sum_{s \in S_1} \max_{x \in A} u(x, s) - \sum_{s \in S_2} \max_{x \in A} u(x, s) \tag{1}$$

holds for all $A \in \mathcal{M}$. Then, we have the following:

Proposition 1. *Every function $U \in \mathbb{R}^{\mathcal{M}}$ is additive. Therefore, every preference on $\mathcal{M}$ admits an additive representation.*

Proof. Fix an arbitrary function $U \in \mathbb{R}^{\mathcal{M}}$. Let $\mathbb{1}$ denote an indicator and define $\varphi \in \{0, 1\}^{2^X}$ by setting $\varphi(A) := \mathbb{1}\{A \neq \emptyset\}$ for each $A \in 2^X$. We claim that there exists a function $\lambda \in \mathbb{R}^{\mathcal{M}}$ such that $U(A) = \sum_{B \in \mathcal{M}} \varphi(A \cap B)\lambda(B)$ for all $A \in \mathcal{M}$. In fact, λ is unique and is explicitly given by the formula $\lambda(B) := \sum_{A \subseteq B} (-1)^{\#(B \setminus A)+1} U(X \setminus A)$ for each $B \in \mathcal{M}$, where # denotes cardinality and we adopt the formal convention that $U(\emptyset) = 0$. This follows from the theory of conjugate Möbius transforms, briefly described in Appendix A.

Define sets $S_1 := \{s \in \mathcal{M} | \lambda(s) > 0\}$, $S_2 := \{s \in \mathcal{M} | \lambda(s) < 0\}$ and a function $u \in \mathbb{R}^{X \times (S_1 \cup S_2)}$ by setting $u(x, s) := \mathbb{1}\{x \in s\}|\lambda(s)|$ for each $x \in X$ and $s \in S_1 \cup S_2$. Then, it is readily verified that $\max_{x \in A} u(x, s) = \varphi(s \cap A)|\lambda(s)|$ and so, for every $A \in \mathcal{M}$, we have

$$\sum_{s \in S_1} \max_{x \in A} u(x, s) - \sum_{s \in S_2} \max_{x \in A} u(x, s) = \sum_{s \in \mathcal{M}} \varphi(s \cap A)\lambda(s) = U(A),$$

proving the first claim. The second claim follows immediately, since every preference on a finite set is representable. □

In the rest of the paper, a pair of sets (S_1, S_2) is a *state-space* (for a fixed U) if there exists $u \in \mathbb{R}^{X \times (S_1 \cup S_2)}$ such that Eq. (1) holds for all $A \in \mathcal{M}$. We call such a function u a *second-stage utility* for state-space (S_1, S_2). Proposition 1 demonstrates that, while Kreps' axioms effectively identify those preferences which can be represented in an additive fashion without resorting to negative states, the additivity per se has no behavioral content. In fact, Proposition 1 shows that *every* representation of a given preference can be written as in Eq. (1).[3] In contrast, Kreps' additivity result without negative states only holds for some monotonic transformation of a given representation.

Proposition 1 tells us that every representation of $\succsim$ has a state-space but yields no further information about its structure. What can be said about the possible state-spaces when $\succsim$ satisfies additional assumptions? The following result describes restrictions imposed by combinations of three standard axioms: set monotonicity, ordinal submodularity and set betweenness. Following Dekel et al. (2009), we divide the set betweenness axiom in two parts: *positive set betweenness* ($A \succsim B$ implies $A \succsim A \cup B$) and *negative set betweenness* ($A \succsim B$ implies $A \cup B \succsim B$).

[3] Using this feature and von Neumann–Morgenstern uniqueness, Proposition 1 can be proven equivalent to Lemma 2 in Ortoleva (2013), a key step for his main results on "thinking aversion". However, the two approaches are very different. Ortoleva's argument is geometrical in nature, associating each lottery over menus of deterministic alternatives with a menu of lotteries and then constructing the representation through an argument similar to DLR's. In contrast, our approach based on Möbius transforms is purely algebraic and more direct. More importantly, it does not involve lotteries at all and, as we will see in Section 3.1, leads to a characterization of minimal state-spaces which allows us to derive restrictions on ex-post behavior.