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Optimal reinsurance with multiple tranches[☆]

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ABSTRACT

Motivated by common practices in the reinsurance industry and in insurance markets such as Lloyd's, we study the general problem of optimal insurance contracts design in the presence of multiple insurance providers. We show that the optimal risk allocation rule is characterized by a hierarchical structure of risk sharing where all agents take on risks only above the endogenously determined thresholds, or agent-specific deductibles. Linear risk sharing between two adjacent thresholds is shown to be optimal when all agents have CARA utilities. Furthermore, we show that the optimal thresholds can be efficiently calculated through the fixed point of a contraction mapping.

1. Introduction

The modern theory of efficient risk sharing goes back to the fundamental paper by Borch (1962), who characterized efficient risk sharing among several agents (typically more than two) with heterogeneous preferences. Based on this research, Wilson (1968) further developed the theory of syndicates. Both Borch and Wilson based their analysis on an important assumption that a complete set of state-contingent contracts is available for risk allocation. In many real-life situations, however, insurers are willing to take only risks that do not exceed a certain level. This situation is particularly true for insurance contracts, for which the corresponding insurance reimbursements (coverage functions) are always assumed to be non-negative and lower than the total loss. As has been shown by Arrow (1971, 1973) and Raviv (1979), such a feature of insurance policy may significantly alter the structure of optimal risk allocation. Namely, the efficient risk-sharing rule *between two agents* (i.e., the insured and the single insurance provider) is generally characterized by the presence of a deductible. The goal of this paper is to extend Raviv's (1979) seminal characterization of optimal insurance design to the case of multiple insurers.

For insurance against loss that can potentially be very large, multiple insurance providers are typically involved to achieve more efficient risk sharing.[1] A well-known example is the so-called *subscription model* at Lloyd's, the world's leading insurance market providing specialist insurance services to businesses.[2] At Lloyd's, almost any single risk is insured by multiple insurers. As is stated on its website, "much of Lloyd's business works by subscription, where more than one syndicate takes a share of the same risk".[3] This is also a well-established practice among insurers generally.[4] Another example of allocating risk among multiple insurance providers is when an insurance company purchases insurance

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1 By distributing large risk across many entities, insurance companies, large and small, can offer coverage limits to meet their policyholders' needs. This is very important for a more competitive insurance market.

2 In his speech on the future of the insurance industry, Lord Levene, the former chairman of Lloyd's, said that "The first point which I want to make about the future of insurance is that the subscription model is not just alive and well—it is thriving. Lloyd's made record profits in 2009. Throughout the financial crisis, it maintained A+ ratings. Over three hundred years, it has never failed to pay a valid claim". Source: http://www.lloyds.com/Lloyds/Press-Centre/Speeches/2011/03/The-Future-of-the-Insurance-Industry.

3 Source: http://www.lloyds.com/lloyds/about-us/what-is-lloyds.

4 See, for example, page 167 of Thoyts (2010) for more discussion.

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from multiple reinsurers.[5] Reinsurance is an indispensable and significant part of the insurance industry and "many reinsurance placements are not placed with a single reinsurer but are shared between a number of reinsurers".[6]

Despite its practical importance, there has been limited amount of research on optimal risk sharing in the presence of more than two insurance providers and the practical constraint that the insurance reimbursement is nonnegative and cannot exceed the size of the loss. The industry practice, which typically involves both proportional and excess of loss contracts with multiple agents offering insurance coverage, seems to be ad-hoc and lacks a strong theoretical basis.[7] This paper fills the gap in the literature by studying the optimal design of insurance contracts with multiple agents offering insurance coverage that satisfies the practical constraints. We also take into account of the intertemporal nature of insurance, which is a realistic aspect given that there is always a (sometimes significant) delay between the insurance premium payment and the arrival of an insurance event. We endogenize this by introducing intertemporal utility maximization for all agents. The framework in this paper applies both to the insurance scenario and the reinsurance scenario. For ease of illustration, we call the agent seeking insurance coverage, whether a client of an insurance company or an insurance company itself, the insured, and the agents offering insurance coverage, whether insurance companies or reinsurance companies, the insurers.

Our first result implies that the practical constraints on insurance contracts, together with insurers' heterogeneity, naturally give rise to *optimal claims splitting through a tranche structure,* with different tranches characterized as the regions for which these constraints are binding for different groups of insurers. The total uncertain loss is divided into several tranches, whose boundaries are the insurer-specific deductibles. Different insurers provide partial coverage for losses inside multiple tranches. This prioritized tranche-sharing structure with multiple deductibles is very intriguing. It arises because of insurers' risk aversion and the heterogeneity of their marginal valuations. The insured optimally insures the first tranche above the minimal deductible with the insurer requiring the lowest marginal premium. Because this insurer is risk averse, the marginal premium increases with the level of losses. Just as the level of losses reaches the next deductible level, the first insurer's marginal premium reaches that of the second-highest ranked insurer, and it becomes optimal for the insured to buy co-insurance of the subsequent tranche from this second-highest ranked insurer. Continuing the process gradually, as the level of losses increases, insurers with higher marginal premia start participating in the trade, until the whole range of loss is exhausted.

To efficiently compute the optimal deductible levels and the co-insurance scheme within each tranche, one needs to compute the endogenously determined minimal marginal rate of intertemporal substitution (MMRIS) of each agent. Our second result is that the insurers' MMRIS can be calculated through the fixed point of an explicitly constructed contraction mapping. This result is crucial, both for the computation of optimal indemnities and for studying the dependence of deductibles on microeconomic characteristics. In particular, we use this result to compute numerical examples of the optimal insurance contracts.

The rest of this paper is organized as follows: In Section 2, we review the relevant literature. In Section 3, we formulate the optimal insurance design problem and characterize optimal indemnities for a finite number of insurers. In Section 4, we show how the optimal contracts can be computed using the fixed point of a contraction mapping and provide several important comparative statics results. In Section 5, we conclude the paper and point out some future research directions. All proofs are in the Appendix.

2. Related literature

This paper extends the classical results of Borch (1962) and Wilson (1968) and can therefore be applied to a large variety of economic problems such as Walrasian equilibrium allocations in complete markets under constraints. In particular, since we allow for heterogeneous discount factors, our results are related to those of Gollier and Zeckhauser (2005), who studied the effect of such a heterogeneity on efficient intertemporal allocations.[8] We show that the practical constraints on insurance contracts together with heterogeneity in discount factors may lead to the failure of classical aggregation results.

In the literature on optimal insurance design, the study most closely related to ours is that of Raviv (1979). He considered the same optimal insurance problem as ours, but with a single insurer and provided necessary and sufficient conditions for the optimality of a deductible. Thus, our results on the optimal insurance design can be viewed as an extension of Raviv (1979) to the case of multiple insurers. In addition, in contrast to Arrow (1971) and Raviv (1979), we also study the intertemporal aspect of optimal insurance design. This allows us to express the optimal allocation in terms of the marginal rates of intertemporal substitution and to link them to various agents' characteristics.

Numerous papers have studied the optimality of deductibles in optimal insurance design in various settings, extending the original model of Raviv (1979). See, for example, Doherty and Schlesinger (1983), Huberman et al. (1983), Blazenko (1985), Gollier (1987, 1996), Gollier and Schlesinger (1995, 1996), Gollier (2004), and Dana and Scarsini (2007). Eeckhoudt et al. (1991) studied the dependence of the optimal deductible on the distribution of losses. Researchers in all of these studies assumed that there is a *single insurer*. The only class of models with multiple insurers that has been extensively studied in the insurance literature corresponds to risk sharing among insurers through a secondary complete capital market, which is not always available in many actual situations. See Aase (2014) for an overview and Citanna and Siconolfi (2015) for more recent development.

Cohen and Einav (2007) and Cutler et al. (2008) found empirical support for the importance of preferences heterogeneity in insurance design and its impact on the optimal deductible choice.

3. The model

The model's participants consist of an insurance buyer (the insured) and a set of N insurance sellers (the insurers). The insurance buyer faces the risk of a random loss, described by a non-negative bounded random variable X with the largest potential loss $\text{esssup} X = \bar{X}$. In addition, the insurance buyer is endowed with other (not explicitly modeled) assets, generating a non-stochastic

5 According to Reinsurance Association of America, "reinsurance is a transaction in which one insurance company indemnifies, for a premium, another insurance company against all or part of the loss that it may sustain under its policy or policies of insurance".

6 Source: http://en.wikipedia.org/wiki/Reinsurance.

7 Reinsurance policies can be categorized according to whether they are proportional or non-proportional with excess of loss contract being the prime example of the latter. An insurance company often purchases several insurance policies of different types from multiple reinsurers and combine these policies to form multiple layers of insurance protection. Chapter 7 of Thoyts (2010) explains with detailed examples how different types of reinsurance policies work.

8 See also a recent paper by Wilson and Kazumori (2009) that studied general efficient intertemporal allocations and extended Wilson (1968) to a dynamic setting.