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Incomplete preferences and confidence☆

Brian Hill *

GREGHEC, HEC Paris, CNRS, Université Paris-Saclay, 1 rue de la Libération, 78351 Jouy-en-Josas, France

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ABSTRACT

A theory of incomplete preferences under uncertainty is proposed, according to which a decision maker's preferences are indeterminate if and only if her confidence in the relevant beliefs does not match up to the stakes involved in the decision. We use the representation of confidence in beliefs introduced in Hill (2013), and axiomatise a class of models, differing from each other in the appropriate notion of stakes. The theory naturally suggests two distinct strategies for completing preferences, and hence for choosing in the presence of incompleteness: one that relies only on beliefs in which the decision maker is sufficiently confident, and one that mobilises all beliefs, no matter how little confidence she may have in them. Axiomatic characterisations are given for completion procedures following each of the strategies. Finally, in a market setting, the incorporation of confidence is shown to add an extra friction, beyond the standard implications of non-expected utility models for Pareto optima.

1. Introduction

Incomplete preferences have been increasingly recognised as of importance. Appeals to the weakening of the completeness axiom—which demands that for every pair of options, the decision maker has a weak preference for one over the other—have been made both in the name of 'psychological realism' (Aumann, 1962; Dubra et al., 2004; Danan, 2003b; Galaabaatar and Karni, 2013) and on the basis of normative considerations (Aumann, 1962; Bewley, 1986/2002). Moreover, incomplete preferences have proved invaluable in the development of alternative models of choice, such as those incorporating a tendency to stick to the status quo (Bewley, 1986/2002; Masatlioglu and Ok, 2005). Incomplete preferences naturally arise in multi-agent settings, where the preferences of a group, or those drawn from group members' beliefs or utilities, may naturally be incomplete (Dubra et al., 2004). As a final example, objectively rational preferences in the sense of Gilboa et al. (2010)—those preferences for which the decision maker can convince others of their correctness, by a form of proof for example—are naturally incomplete.

The traditional approach to modelling incomplete preferences proceeds, roughly speaking, by dropping the completeness axiom whilst retaining the other standard axioms, and replacing the single function or measure in the relevant model by a set. For instance, in decision under uncertainty, the benchmark unanimity multi-prior model proposed by Bewley (1986/2002) retains all standard Anscombe and Aumann (1963) axioms for subjective expected utility except completeness, and replaces the single probability measure in the representation by a set of probability measures. In particular, it retains the independence axiom.

However, under all of the interpretations mentioned above, there appear to be cases where the standard independence axiom is violated. Consider a decision maker who is faced with choices between bets on the colour of the next ball drawn from an urn containing only black and white balls, as shown in Fig. 1. For simplicity, suppose that the bets are given in dollars and the decision maker has linear utility.[1] She is told neither the proportion nor the number of balls in the urn, but she has observed fifteen draws (with replacement), nine of which were black and the rest of which were white. It does not seem implausible that there are decision makers who prefer f to 0 given this information, whilst being indeterminate in their preference between g and 0. Certainly, from a normative point of view, it is not unreasonable to hold a preference between the first pair of bets while not

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* Fax: +33 1 39 67 71 09.
E-mail address: hill@hec.fr.

[1] Alternatively, one could read the bets as given in utils, and as corresponding to the appropriate mixtures of corresponding dollar bets in the standard way; e.g. f is the mixture $\frac{1}{100\,000}g + \frac{99\,999}{100\,000}0$.

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	Colour of ball drawn from urn	
	Black	White
f	15	-10
0	0	0
g	1.5 M	-1 M
f^n	$15 \times n$	$-10 \times n$

Fig. 1. Bets ('M' stands for 'million').

having a determinate preference between the second pair, given the weakness of the information and the stakes involved. Even from the point of view of objective rationality, there is a 'statistical argument' for preferring f over 0—based, for example, on a classical hypothesis test with a weak significance level (e.g. 10%)[2]—whereas there is no objectively rational preference between g and 0—in the situation where more is at stake, arguably more stringent standards of proof, such as tougher significance levels, are required, and the data do not support any conclusions at such levels. Analogous cases exist for the group interpretation of incomplete preferences: for example, if there is agreement between two leading urn-experts that the proportion of black balls is $\frac{1}{2}$, but a large disagreement in the community as a whole on the proportion of black balls, it is does not seem unreasonable for the group to form a preference between f and 0 without forming one between g and 0. Since independence implies that there is preference for f over 0 if and only if there is preference for g over 0, it is violated in these examples.

Reinterpreting the event that the ball is black to be the success of a new technology, for example, and the observations to be suggestive yet inconclusive findings, it is clear that there are real-life cases where this sort of preference pattern is exhibited. On the basis of limited grounds (be they scarce information, a weak argument or agreement among a few members of the group), decision makers may be ready to form preferences when the decision is relatively unimportant, but cannot do so when there is more at stake. Our proposed diagnosis is that the traditional models of incomplete preferences (in terms of sets of probability measures, for example) overlook the fact that decision makers can be more or less *sure* of their beliefs. The examples given above suggest that *how* sure the decision maker is in a belief may be related to her preferences over options for which this belief is relevant. These appear to be cases where determinate preferences are formed on the basis of beliefs in which the decision maker is not entirely sure in some situations—in particular, when little is at stake in the decision—whereas there are other situations—when the decision is more important, for example—in which she may need to be more sure of her beliefs to avoid indeterminacy.

The aim of this paper is to propose a model of decision under uncertainty that, whilst deviating as little as possible from standard models of incomplete preference, incorporates the decision maker's confidence in her beliefs. Inspired by the above considerations, it seems that an appropriate model should adhere to the following maxim: one's preferences are indeterminate when and only when one's confidence in the beliefs needed to form a preference does not match up to the stakes involved in the choice. We develop such a model, drawing on existing research on confidence in belief and its role in decision making, and in particular on the concepts introduced in Hill (2013). Like the standard Bewley model, we focus on indeterminacy of preferences that is driven solely by the decision maker's beliefs, tacitly assuming that she is fully confident in her utilities.

As concerns behavioural properties, note that in the context of incomplete preferences, independence applied to the preference $f \succ 0$ and the acts g and 0 (Fig. 1) in fact implies two distinct things: on the one hand, there is a determinate preference between g and 0; on the other hand, this preference goes in the appropriate direction ($g \succ 0$). The examples above only conflict with the former condition, not the latter; however, it is the latter condition that is at the heart of the independence property. Hence it is natural to drop the former condition, retaining the latter: that is, to demand that the standard independence condition applies whenever the preferences involved are determinate. This is the appropriate weakening of independence for the model developed in this paper. Indeed, the other main axiomatic difference from the Bewley multi-prior model involves a similar weakening of transitivity: it applies whenever preferences are determinate, but indeterminacy is permitted in some cases where standard transitivity would have demanded determinate preference.[3] We take the mildness of these axioms to be an indication of the parsimony of this departure from the benchmark Bewley model of incomplete preferences under uncertainty.

Another central contribution of the paper is to identify some interesting consequences of the incorporation of confidence for the question of how to 'complete' preferences—a question that is pertinent under all the aforementioned interpretations, in particular when a decision must be taken. It allows the distinction between, and characterisation of, two strategies for preference completion. One respects confidence, insofar as it only allows the decision maker to use beliefs in which she has sufficient confidence given the stakes involved in the decision. A government who bases its climate policy on 'full scientific certainties', however scarce they may be and ignoring the less well-established opinions of experts, adopts this strategy. The other strategy goes on hunches, insofar as it allows the decision maker to mobilise all her beliefs—even those in which she has little confidence—when she is forced to choose. An entrepreneur who undertakes a venture on the basis of her 'gut feeling', without being strongly convinced of its success, is adopting this strategy. The distinction between these strategies, though pre-theoretically reasonable and potentially pertinent to the understanding of real-life decisions, has not yet been identified in the literature, to our knowledge.

Finally, a standard interpretation of indeterminacy of preferences in market settings (dating back at least to Bewley, 1986/2002) is in terms of reluctance to trade, and it is natural to ask what implications the incorporation of confidence into models of incomplete preference has in such settings. We show that it adds a friction absent under other non-expected utility or incomplete preference models of decision under uncertainty, with consequences for the difficulty of attaining a Pareto optimum via Pareto-improving trade.

The basic notions of the model are introduced and formally defined in Section 2. The model is formally stated in Section 3.1, and the representation result is given in Sections 3.2–3.3. Section 3.4 contains a comparative statics analysis. In Section 4, we consider the question of how to complete one's incomplete preferences. In Section 5, we consider the consequences of the model in markets under uncertainty. Related literature is discussed in Section 6. Proofs of all results and other material are to be found in the Appendices.

[2] Explicitly, a one-sided classical statistical test rejects the hypothesis that the proportion of black balls is 0.4 at the 10% significance level, and for probabilities of black above 0.4, f has a higher expected utility than 0.

[3] The need for a weakening of transitivity can also be seen on the example above. It is not implausible, in the light of similar considerations to those behind the preference for f over 0, that the decision maker prefers f^{n+1} to f^n for all n between 0 and 99 999 (recall that she has linear utility). Transitivity would imply that she prefers g over 0, and hence is violated. See Section 3.2 for further discussion.