Journal of Mathematical Economics ▮ (▮▮▮▮) ▮▮▮–▮▮▮

Contents lists available at ScienceDirect

Journal of Mathematical Economics

journal homepage: www.elsevier.com/locate/jmateco

Robust bubbles with mild penalties for default☆

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ARTICLE INFO

Article history:
Available online xxxx

Keywords:
Bubbles
Self-enforcing debt
Inside liquidity
Outside liquidity
Endogenous debt limits
Risk-sharing

ABSTRACT

Limited enforcement of debt contracts and mild penalties for default can lead to low equilibrium interest rates, to ensure debt repayment. Low interest rates, in turn, create conditions for bubbles. I show that bubbles in unsecured private debt exist when the punishment for default is a permanent or a temporary interdiction to trade. Bubbles are an inefficient source of liquidity, as they lower interest rates and reduce welfare by discouraging saving.

1. Introduction

A rational bubble is defined as the price of an asset in excess of the present value of its dividends. Santos and Woodford (1997) show that bubbles on assets in positive supply cannot exist if the interest rates are high, that is, if the present value of aggregate endowment is finite.[1] Bubbles grow on average at the same rate as the interest rates and therefore they are positive martingales when discounted by the pricing kernel. With high interest rates, a bubble would become too large relative to the aggregate endowment.

This paper shows that low interest rates arise naturally to induce repayment in economies with limited enforcement of debt contracts, and this can lead to bubbles. Agents have the option to default on debt and incur a punishment, leading to a (reduced) continuation utility that can be date and state dependent. As in Alvarez and Jermann (2000), markets select endogenously the largest debt limits for the agents so that repayment is always individually rational given future bounds on debt. Thus the credit made available to a borrower depends on the value of an unblemished reputation. Bubbles here refer to the existence of martingale components in agent's credit limits. They are a form of *inside liquidity*, being part of the unsecured debt obligations issued by financially constrained consumers (within the consumption sector). As it will be shown, these martingale components in debt limits can be substituted by fiat money (or by unbacked public debt), justifying the use of the term "bubble".

I document the existence of bubbles for some of the most common penalties for default encountered in the literature: a permanent *interdiction to trade* (IT) (Kehoe and Levine, 1993, 2001, or Alvarez and Jermann, 2000, 2001), a temporary *interdiction to trade for a finite* and deterministic number of periods (ITF) (Azariadis and Kaas, 2008), or an *interdiction to trade for a random* number of periods (ITR) (Azariadis and Kaas, 2013). Under penalty (IT), a defaulting agent is permanently excluded from the markets and consumes his endowment. With penalty (ITF), after default, an agent cannot trade for a predetermined number of periods, after which full trading privileges are restored. Finally, under penalty (ITR), an agent in default permanently regains full access to the markets with some fixed probability per period.

Hellwig and Lorenzoni (2009) were the first to uncover the existence of bubbles in unsecured debt, for the case when the punishment for default is an *interdiction to borrow* (IB).[2] Any non-autarchic equilibrium must have low interest rates, and agents' endogenous discounted debt limits are martingales. They conclude that empirical evidence in favor of low interest rates and bubbles in debt would represent a confirmation of the particular reputational mechanism (IB) (Hellwig and Lorenzoni, 2009, p. 1156).

The results of this paper indicate that the presence of low interest rates and bubbles is not driven by the particular penalty (IB). Moreover, the policy implications of bubbles under the penalties studied here are dramatically different from those obtained by Hellwig and Lorenzoni (2009) for (IB). In their model,

☆ Particular instances of some of the results appeared in Bidian (2011, Chapter 4).

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1 For deterministic economies, their results were anticipated by Kocherlakota (1992) and later refined by Huang and Werner (2000). The results of Santos and Woodford (1997) were extended to economies with general portfolio constraints and differential information by Bidian (2011) and Bidian (2014).

2 Under this penalty, defaulting agents can continue to save after default. Their example is discussed also in Werner (2014).

http://dx.doi.org/10.1016/j.jmateco.2015.04.002
0304-4068/

due to the weaker penalty for default (IB), trade can only be sustained under low interest rates, to induce borrowers to repay debt. Bubbles make trade possible (prevent autarchy) and bigger bubbles lead to more risk-sharing and trading. Under the harsher penalties considered here, trade can occur without bubbles, in Pareto-dominating equilibria with high interest rates. Bubbles can lead to inefficiently low interest rates, deterring saving and risk-sharing. Their size is not comonotonic with the amount of risk-sharing and trade in the economy.

Kocherlakota (2008) proved that an arbitrary bubble can be injected in the price of an asset, while preserving consumption and real interest rates, as long as agents' debt limits are tightened by the bubble times their initial endowment of the asset (sterilizing the wealth effect of the bubble injection).[3] In order for the tighter debt limits to remain nonpositive, the initial debt limits had to have martingale components. The example of Hellwig and Lorenzoni (2009) was the only known economy with martingale components in debt limits, and therefore the only known example where the bubble injection mechanism of Kocherlakota (2008) does not lead to positive debt limits. This paper identifies additional environments where martingale components are present in debt limits, which can be converted into asset price bubbles (valued fiat money).

The unifying thread within the literature on rational bubbles in either endowment or production economies is the presence of financial frictions (portfolio constraints). Bubbles help relax these constraints, and therefore agents willingly hold the bubbles (overvalued assets). The early models focused on overlapping generations (OLG) models, which have (implicit) built-in portfolio constraints. Indeed, old agents cannot borrow (against the income of their offsprings), and they have to save when young. A high demand for saving puts downward pressure on interest rates and creates conditions for bubbles. Tirole (1985) shows that bubbles, by serving as an additional store of value, reduce the (inefficient) overaccumulation of capital, raise interest rates and increase welfare. Newer OLG models add explicit financial frictions in the form of pledgeability limitations or scarce collateral (Caballero and Krishnamurthy, 2006; Farhi and Tirole, 2012; Giglio and Severo, 2012; Martin and Ventura, 2012). Bubbles now can crowd-in investment and emerge even in dynamically efficient economies.

With infinitely lived agents, explicit portfolio constraints are needed to prevent Ponzi schemes. If these constraints do not bind, bubbles are ruled out by agents' transversality conditions. Intuitively, optimizing agents do not allow for their financial wealth to become too large relative to their marginal utility of consumption. Binding constraints decouple the interest rates from an agent's intertemporal marginal rates of substitution. The agent might willingly hold a bubble growing on average at the rate of interest rates, because his marginal utility decreases faster, allowing for his transversality condition to be satisfied.[4] In fact, models with infinitely lived agents that are subject to occasionally binding portfolio constraints can behave very similarly to OLG models (Woodford, 1990). Such constraints effectively segment the horizons of the agents as in OLG models, and bubbles can arise to alleviate the underlying frictions. The constraints can be exogenously given (Kocherlakota, 1992; Huang and Werner, 2000; Santos and Woodford, 1997), can reflect a lack of safe stores of value (Aoki et al., 2014), or can be endogenized by limited pledgeability or limited collateral (Kiyotaki and Moore, 2008; Kocherlakota, 2009; Hirano and Yanagawa, 2010; Miao and Wang, 2011). Among these papers, Miao and Wang (2011) is the only one in which the bubble is attached to a productive asset (firm) generating endogenous dividends as a result of optimal production decisions, rather than being simply valued fiat money.[5]

This paper, together with Hellwig and Lorenzoni (2009), are unique in their focus on rational bubbles in the unsecured debt (sustained by reputation) arising within the consumption sector (for example, credit card debt). In Hellwig and Lorenzoni (2009), where the penalty for default (IB) is weak, bubbles make trade possible,[6] raise interest rates (above the autarchic level), improve welfare and can result in (constrained) efficient allocations. Under the more severe penalties for default studied here (milder financial frictions in the form of enforcement limitations), novel insights emerge. Trade can be sustained without bubbles. In these situations, bubbles provide liquidity *inefficiently*. In contrast to most of the literature on rational bubbles, they *lower* the interest rates,[7] deter saving and *decrease* welfare. Bubbles are made possible by self-fulfilling expectations of tight future credit conditions, which reduce the reputational value of not defaulting in the current period. Default is prevented by a drop in interest rates and in current credit.

The potential for bubbles to reduce welfare was known in models with endogenous growth, where bubbles can lower the growth rate of the economy. Grossman and Yanagawa (1993) introduce externalities in capital, which leads to underinvestment rather than overinvestment in capital. By further crowding out capital, fiat money reduce growth. Olivier (2000), however, shows that their conclusion is not robust if, instead of fiat money, the bubble is attached to productive assets (equity). In general, bubbles crowd out the accumulation of (knowledge) capital, but increase the interest rates and the return on innovation, stimulating it. If the first (second) effect dominates, bubbles decrease (increase) growth (Tanaka, 2011).[8]

The most severe penalty I analyze is (IT), as harsher penalties rule out bubbles. Indeed, if creditors can confiscate an arbitrarily small fraction of the current and future income of defaulting agents, in addition to banning them from the markets, Bloise et al. (2013, Appendix B) show that high interest rates must arise in equilibrium. The reason is that each agent's debt limits, in absolute value, bound from above the present value of the fraction of agent's endowment that is garnished upon default. Penalty (IT) was also analyzed by Bloise et al. (2013) and Antinolfi et al. (2007), in a deterministic version of the (stochastic) model considered here. However, their characterization of equilibria is incomplete, as they have not calculated the endogenous debt limits and establish the existence of martingale components in these debt limits.

It is important to notice that under all the punishments for default and parameters allowed here, *autarchy* is an equilibrium with low interest rates, supported by zero debt limits, which cannot sustain bubbles. Hence low interest rates are only a necessary (rather than sufficient) condition for bubbles. One needs to characterize the endogenous debt limits for the various penalties

3 He establishes this result for environments with complete markets, and refers to it as the "bubble equivalence theorem". The result holds also with incomplete markets, in a qualified form (Bejan and Bidian, 2014).

4 For a derivation of the necessary and sufficient transversality conditions for an optimizing agent facing debt constraints, see Bidian and Bejan (forthcoming).

5 Miao and Wang (2012, 2014, 2015) enrich the model of Miao and Wang (2011) along various dimensions. Miao (2014) is a recent survey of the literature on rational bubbles. Brunnermeier and Oehmke (2013) review also non-rational (behavioral) bubbles.

6 Without bubbles, the enforcement limitations prevent any trade.

7 That bubbles raise interest raise is a rather undisputed feature of bubbles, as argued by Farhi and Tirole (2012). The sole exception until now was Martin and Ventura (2012), where the creation of bubbles by new productive entrepreneurs could lower interest rates.

8 Miao and Wang (2014), in a two-sector model of endogenous growth, show that bubbles can also increase or decrease growth, depending on which sector they occur. Caballero and Krishnamurthy (2006) showed that bubbles can reduce welfare in an open economy context.

Please cite this article in press as: Bidian, F., Robust bubbles with mild penalties for default. Journal of Mathematical Economics (2015), http://dx.doi.org/10.1016/j.jmateco.2015.04.002